Thermal characterisation of groundwater systems in heterogeneous alluvial sediments: Insights from field tests in Hronsek (Slovakia)

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Abstract: This study investigated the thermal parameters in a heterogeneous alluvial aquifer in Hronsek (Slovakia), with the aim of evaluating the suitability of analytical models for interpreting thermal response test data under groundwater flow. We conducted pumping tests and various thermal response tests in a central well and multilevel temperature monitoring in observation wells. Data were analysed by fitting observations to several analytical solutions, specifically the finite line source model, moving infinite line source with dispersion model, and moving finite line source model, using a grid search algorithm to estimate the bulk thermal conductivity and volumetric heat capacity. The results consistently showed significant subsurface heterogeneity, indicated by distinct curves in diagnostic time-over-radius squared plots derived from both hydraulic and thermal tests. The extended conduction-dominated thermal response test successfully produced a strong and detectable thermal signal. The best-fit parameters varied significantly between the analytical models. The Moving Finite Line Source model provided the best overall fit with thermal conductivity 9.4 $W.m.K^{-1}$ and volumetric heat capacity 7.0 × 10^6 $J.m^{-3}.K^{-1}$. The thermal conductivity values of the Finite Line Source and Moving Finite Line Source models were notably higher than the typical literature values for saturated sandy gravels. Furthermore, non-uniqueness of parameters was observed, where different combinations of parameters produced similar model-data discrepancies. The study concludes that standard analytical models face limitations in accurately characterizing thermal properties in heterogeneous aquifers with active groundwater flow, emphasizing the need for multifaceted experimental designs and sophisticated modeling approaches for reliable thermal characterization.

Keywords: thermal properties of the aquifer, thermal response test, heterogeneous medium, groundwater flow, thermal conductivity, volumetric heat capacity

1. INTRODUCTION

The thermal regime of groundwater systems is a critical factor in various hydrogeological and geo-energy applications. These include geothermal energy extraction, managed aquifer recharge, and assessment of impacts of climate change on subsurface environments. Therefore, understanding the key thermal parameters of alluvial sediments, such as thermal conductivity (λ) and volumetric heat capacity (Cm), is essential for the effective design and management of these systems. Thermal conductivity quantifies a material's ability to conduct heat, whereas the volumetric heat capacity represents its capacity to store thermal energy per unit volume.

Estimating the effective thermal conductivity and thermal capacity in shallow aquifers with groundwater flow involves understanding the various factors that influence and regional variations. The thermal properties of aquifers are significantly affected by groundwater flow, which increases geothermal potential through advection, as demonstrated in studies of alluvial aquifers, where fast-moving groundwater increases energy replenishment (Previati & Crosta, 2024). Advection refers to the process of heat transfer through the bulk movement of a fluid, such as groundwater. This process contrasts with conduction, which is the transfer of heat through direct contact between molecules and is typically dominant in stationary media.

Thermal Response Tests (TRTs) are commonly employed field methods for estimating subsurface thermal properties. These tests typically involve injecting heat into a well and monitoring the resulting temperature variations. When conducted in aquifers with significant horizontal groundwater flow, TRTs can provide valuable insights into hydraulic conductivity (Wagner et al., 2014). The thermal conductivity of the subsurface is a critical parameter for designing ground-source heat pump systems (GSHPs), and its accurate estimation often involves methods that account for regional geological variations and uncertainties (Heim et al., 2022). For example, regional studies such as those in the North China Plain illustrate how thermal conductivity varies across different hydrogeological regions (Wang et al., 2022). The profound influence of groundwater flow on thermal conductivity is further supported by experimental investigations that show a direct correlation between flow velocity and increased effective thermal conductivity in various types of sediment (Huber & Arslan, 2015).

However, the presence of flowing groundwater significantly complicates the determination and interpretation of these thermal parameters of the TRTs. Advective heat transport, driven by groundwater flow, can overshadow conductive processes,

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making standard TRTs and their interpretation more challenging. Traditional analytical solutions used to interpret TRT data, such as the infinite line source (ILS) model and its modifications (e.g., Cooper-Jacob, FLS, MILS, MILSd, MFLS as referenced by Stauffer et al. (2014), typically assume homogeneous and isotropic subsurface conditions and predominantly conductive heat transfer. Although effective in many geological settings, the accuracy of these analytical solutions can be significantly compromised in aquifers with significant groundwater flow, where advection becomes a dominant heat transport mechanism. Furthermore, natural geological formations are rarely homogeneous; alluvial sediments often exhibit significant heterogeneity in terms of grain size distribution and hydraulic properties, which directly impact thermal properties and their spatial variability. Ignoring advection and heterogeneity can lead to considerable errors in the estimation of thermal parameters, subsequently affecting the design and efficiency of ground-source heat pump systems or other thermal applications.

The limitations of existing analytical methods in characterising thermally complex and advection-dominated groundwater systems are particularly evident. Although some analytical solutions attempt to incorporate advective effects, they often simplify the complex interplay between conduction, advection, and subsurface heterogeneity. Preliminary studies conducted in Hronsek (Slovakia), for example, highlighted that none of the analytical methods previously applied provided a fully satisfactory fit to the data obtained from field tests in a heterogeneous sandy gravel aquifer. This discrepancy underscores a critical research challenge: standard techniques may not adequately capture the thermal behaviour in systems where groundwater flow and geological variability are pronounced. Regional studies, such as those conducted in Denmark, consistently highlight the importance of local thermal conditions in the design and efficiency of geothermal installations, highlighting the need for comprehensive thermal property assessments (Møller et al., 2020). Furthermore, integration of hydrological, thermal, and geophysical data through advanced inversion schemes allows high-resolution estimation of subsurface properties, which is crucial to understand water and heat dynamics in shallow aquifers (Tran et al., 2016). In general, effective thermal management of aquifers, particularly in urban areas, requires integrated spatial planning and consideration of both natural and anthropogenic heat sources to optimise the utilisation of geothermal resources (Bayer et al., 2023).

Given these complexities, there is a clear and pressing need to investigate alternative or refined methodologies and to understand the sensitivity of parameter estimates to different interpretative approaches, especially when dealing with the inherent complexities of real-world alluvial aquifers.

This study aims to address these challenges by investigating and comparing different techniques to measure and interpret thermal parameters in an alluvial aquifer characterised by significant groundwater flow and subsurface heterogeneity. Through a series of field tests, including pumping tests,

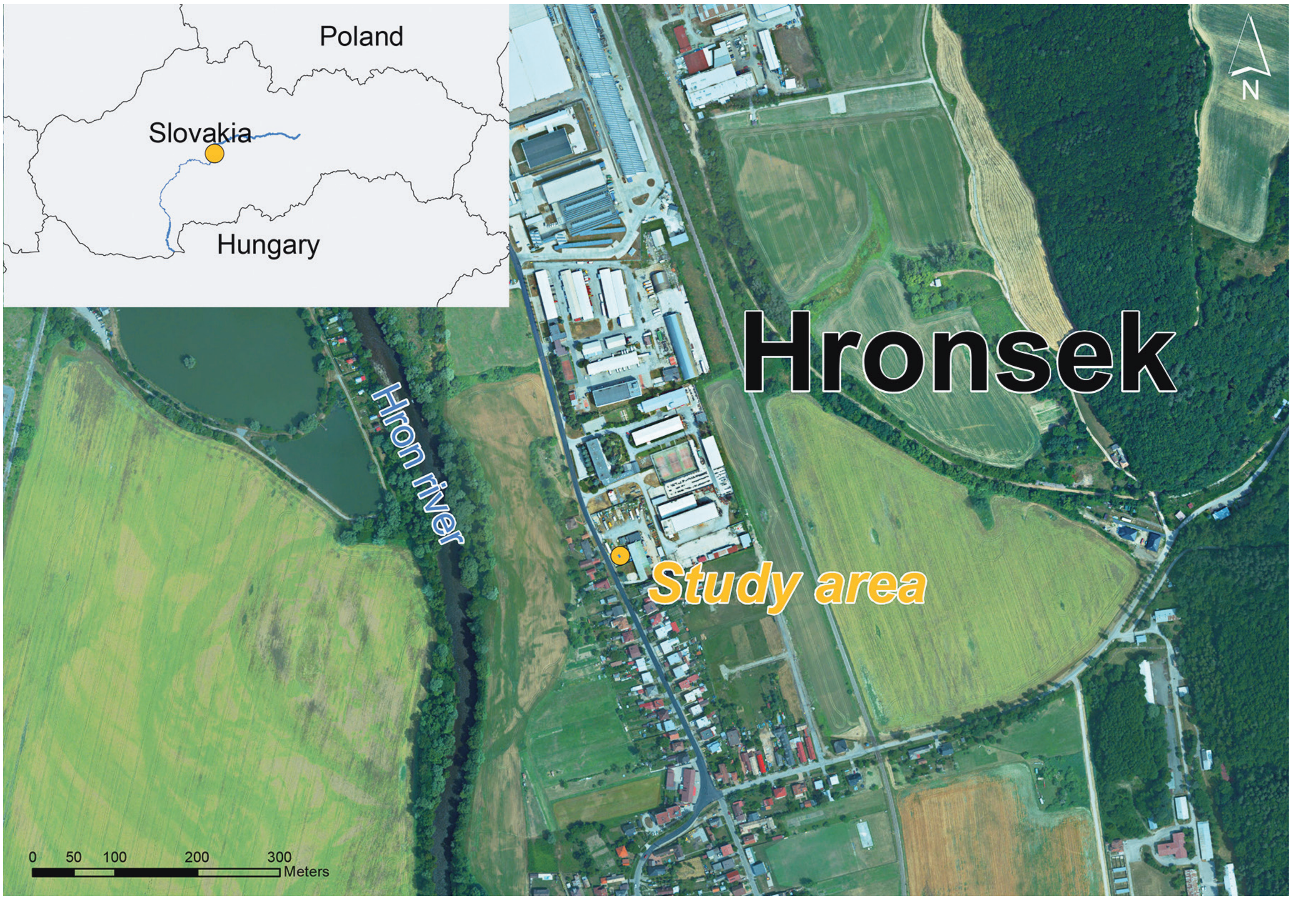


Fig. 1: Location of the study site.

conduction-dominated heating tests, and advection-dominated heating tests that involve infiltration of heated water, this research evaluates the efficacy of various analytical solutions. The core objective is to assess the limitations of these methods and explore the impact of heterogeneity on parameter estimation, ultimately seeking to identify more reliable approaches for characterising the thermal properties of such complex groundwater systems. The findings are critical for improving the accuracy of thermal site characterisation, which is fundamental for the successful implementation of thermal energy projects in the subsurface and for sustainable groundwater resource management.

2. METHODS

This study used a field-based experimental methodology to investigate the thermal parameters of an alluvial aquifer system characterised by flowing groundwater and significant heterogeneity. The core approach involved conducting a series of in situ hydraulic and thermal tests, including pumping tests and multiple types of thermal response tests (TRTs), designed to capture different dominant heat transport processes. This methodology was chosen because laboratory measurements on core samples, while useful, often fail to represent bulk thermal properties at the field scale, especially in heterogeneous formations where preferential flow paths and large-scale structures influence thermal transport. Field tests allow for the evaluation of effective thermal parameters under real-world conditions.

The justification for conducting multiple types of heating tests (one focussing on conduction and another on advection) stems from the known complexity introduced by groundwater flow, which can render purely conductive models inadequate. By inducing different thermal regimes, we aimed to better specify and understand the contributions of both conductive and advective heat transport. Furthermore, the application and comparison of various established analytical solutions to interpret the TRT data were performed to assess their suitability and limitations in this specific hydrogeological context, which is suspected to deviate from idealised homogeneous conditions.

The field investigation was carried out at a site in Hronsek, Slovakia (Fig. 1).

The study area is characterised by an unconfined alluvial aquifer composed primarily of heterogeneous sandy gravel and sand, with an approximate thickness of 6 metres. This aquifer is directly underlain by Neogene clays, forming a lower aquitard. The ground surface at the site is approximately 311.65 metres above sea level (m a. s. l.), and the base of the aquifer is situated at around 305.60 m a. s. l.

Key hydraulic parameters for the site were previously estimated. The natural hydraulic gradient is approximately 0.01 $m.m^{-1}$. The mean groundwater head was observed at 309.48 m a. s. l., and the average hydraulic conductivity of the aquifer material was estimated to be 1.6×10^{-3} $m.s^{-1}$. The general direction of groundwater flow was determined before testing (Fig. 2). These baseline conditions, particularly the notable hydraulic conductivity and gradient, suggested that advective heat transport would likely be a significant factor in the thermal regime of the aquifer.

The testing site for this study was the in situ alluvial aquifer itself, specifically the volume of sediment and groundwater influenced by the induced hydraulic and thermal stresses. The investigations focused on a heated well (designated as well 3) and a network of four multilevel observation wells (wells 3, 4A, 4B, 4C). These observation wells were strategically positioned to monitor the propagation of thermal and hydraulic signals in three dimensions.

To directly assess the physical characteristics and heterogeneity of the aquifer material, core samples were extracted from well 3 (Fig. 3). These samples provided a visual representation of geological variability within the aquifer, confirming its heterogeneous nature consisting of sandy gravel and sand. The multilevel observation wells were designed to capture data at different depths within the aquifer, allowing for an assessment of vertical variations in the temperature response.

The field experiments involved a suite of specialised equipment and carefully executed procedures.

Heating system: Thermal energy was introduced into the aquifer using an electrical heater with a maximum power input of 2.5 kW. This heater was installed in depth from 5.9 to 6.6 meters below surface within the designated heated well 3.

Monitoring equipment: Temperature changes within the heated well and the surrounding observation wells were monitored using Fiedler dataloggers equipped with six Pt100 thermistors each, allowing multilevel temperature measurements (in depths 6,5 m; 5,5 m; 4,5 m; 3,5 m and 2,5 m below surface). In

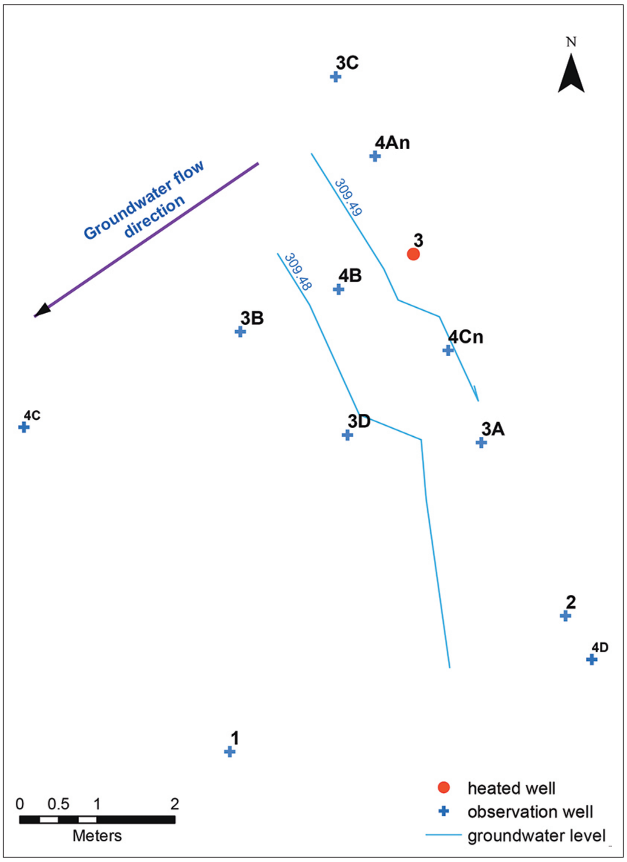


Fig. 2: Map displaying heated well 3, observation wells and groundwater flow direction.

Fig. 3: Heterogeneous material in the core sample of well 3. Photo: D. Krčmář

addition, Solinst dataloggers (4 m below surface) were used in all observation wells for continuous recording of temperature and water levels. Figure 4 shows the electric heater, its installation, and the data recording equipment.

Field test procedures:

1. **Pumping test:** A standard pumping test was conducted to determine the aquifer's hydraulic parameters (transmissivity and storativity). This involved pumping water from a well and monitoring drawdown in observation wells over time, followed by a recovery period.
2. **Heating test (conduction-dominated):** This test involved activating the electrical heater in well 3 to generate a thermal pulse. Pumping was likely minimised or controlled to allow heat to dissipate primarily through conduction, providing data for comparison with scenarios where advection is more prominent.
3. **Heating test by infiltration of heated water (advection-dominated):** An innovative approach was employed where heated water was actively infiltrated into the aquifer. This test likely involved simultaneous water extraction to create a hydraulic depression, thereby inducing a controlled flow field for the heated water. This procedure was specifically designed to emphasise and study advective heat transport mechanisms.

- **Data acquisition:** Continuous data recording of temperature and water levels was performed throughout all tests at multiple depths in the observation wells to capture the temporal and spatial response of the aquifer system.

The data collected from field tests were subjected to several analytical techniques:

- **Pumping test analysis:** Data from the pumping test (drawdown and recovery) were analysed using established methods such as Theis (1935) and Cooper & Jacob (1946) solutions to estimate the transmissivity of the aquifer (T) and the storage coefficient (S). These methods are standard for interpreting hydraulic tests in porous media.
- **Heating test analysis:** Thermal data (temperature changes over time) from conduction and advection-dominated heating tests were analysed to estimate the thermal conductivity (λ) and the volumetric heat capacity (C_m) of the aquifer. This was achieved by fitting the observed data to several analytical solutions commonly used for TRT interpretation, including the Infinite Line Source (ILS) model and its variants such as the Infinite Cylindrical Source (ICS), Finite Line Source (FLS), Moving Infinite Line Source (MILS), Moving Infinite Line Source with delayed start (MILSd), and Moving Finite Line Source (MFLS), as described by Stauffer et al. (2014).
- **Parameter estimation and model fitting:** An automatic fitting procedure was used employing a grid search algorithm to find the optimal set of thermal parameters (λ and C_m) that minimised the discrepancy between the predictions of the analytical model and the measured temperature data. The goodness of fit for each analytical model was quantified using

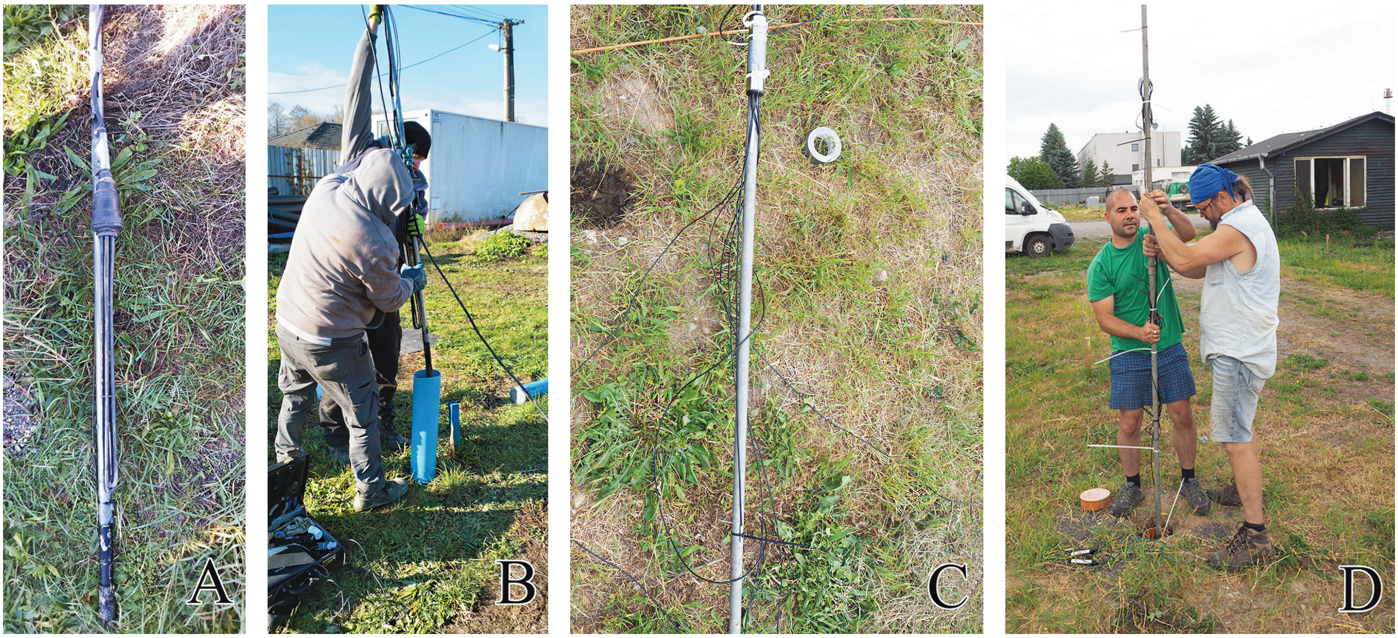


Fig. 4: A) Electric heater; installation of heather. B) Fiedler dataloger with Pt100 termistor. C) installation of datalogger. D) Solinst datalogers. Photo: D. Krčmář

the root mean square error (RMSE).

- **Heterogeneity assessment:** A semi-logarithmic graph of t.r^{-2} (time divided by the square of the radial distance from the heat source) was used as a diagnostic tool. Deviations from a straight line on such plots can indicate subsurface heterogeneity or deviations from purely conductive heat flow, helping to qualitatively assess the complexity of the system.

The choice to apply multiple analytical models and a grid search for fitting was justified by the anticipated complexity of the site and the objective of rigorously evaluating which models best represent the thermal behaviour in such a heterogeneous, advection-influenced system.

2.1. Heating test analysis and analytical modelling

Thermal data (temperature changes over time, ΔT) from the heating tests were analyzed to estimate the bulk thermal conductivity (λ) and volumetric heat capacity (Cm) of the aquifer. This was achieved by fitting the observed ΔT data to several analytical solutions, as described by Stauffer et al. (2014), which are commonly used for interpretation of thermal response tests (TRT). Specific models implemented and evaluated in this study using custom Python scripts include the Finite Line Source (FLS), Moving Infinite Line Source with Dispersion (MILSd), and Moving Finite Line Source (MFLS) models.

The theoretical basis for each implemented model is as follows (all based on work Stauffer et al. (2014):

1. **Finite line source (FLS) model:** The FLS model (e.g., based on Kelvin's line source theory, extended for finite length) calculates the temperature change at an observation point due to a line source of finite length releasing heat at a constant rate per unit length (Q_L). This model assumes purely conductive heat transport in a homogeneous and isotropic medium. The temperature increases Δ_T at a radial distance r (calculated from coordinates x, and the and time t is determined by integrating the point source solution along the length of the heater. The Python script implements this by calculating thermal diffusivity ($D_t = \lambda/\text{Cm}$) and then numerically integrating an expression involving the complementary error function (*erfc*) over the heater length. The general form involves the following:

$$\Delta T(r,z,t) = \frac{Q_L}{4\pi\lambda}\int_{\text{source}} \frac{\operatorname{erfc}\left(\frac{\sqrt{r^2+(z-z')^2}}{2\sqrt{D_t t}}\right)}{\sqrt{r^2+(z-z')^2}}\,\mathrm{d}z' \qquad (1)$$

where the integral is taken over the source length. The script specifically implements this using a method of images or a specific boundary condition representation:

$$\Delta T(r,z,t) = \frac{Q_L}{4\pi\lambda}\left(\int_0^H f(z',r,z,D_t,t)\,\mathrm{d}z' - \int_{-H}^0 f(z',r,z,D_t,t)\,\mathrm{d}z'\right) \qquad (2)$$

where is the integrand from Equation 1. The parameters for groundwater velocity (v_T) and dispersivities (a_x, a_y) are present in the function signature in the script but are effectively set to zero or negligible values, confirming a purely conductive approach for this FLS implementation.

2. **Moving infinite line source with dispersion model (MILSd):** The MILSd model, extends the infinite line source concept to account for advective heat transport due to groundwater flow (velocity v_T) and hydrodynamic dispersion. The model assumes that the heat source is infinite in vertical dimension. The temperature increase is influenced by thermal conductivity (λ), volumetric heat capacity (C_m), groundwater velocity (v_T), and longitudinal (α_l) and transverse (α_T) dispersivities, which contribute to the dispersion coefficients $D_x = D_t + \alpha_T v_T$ and $D_y = D_t + \alpha_T v_T$ with $D_t = \lambda / C_m$.
The Python script calculates the temperature change using an analytical solution. The equation implemented in the script is:

$$\Delta T(x,y,t) = \frac{Q_L}{4\pi C_m\sqrt{D_x D_y}}\exp\left(\frac{v_T x}{2D_x}\right)\times \int_{\frac{x^2}{4D_x t}+\frac{y^2}{4D_y t}}^{\infty} \frac{1}{\psi}\exp\left(-\psi - \frac{v_T^2}{16 D_x \psi}\left(\frac{x^2}{D_x}+\frac{y^2}{D_y}\right)\right)\mathrm{d}\psi \qquad (3)$$

The parameters H (source length) and z (vertical position) present in the function signature is not used in the core MILSd calculation, consistent with an infinite line source assumption.

3. **Moving finite line source (MFLS) model:** The MFLS model extends the classical line source concept by considering a finite-length heat source moving with groundwater flow, thus combining conductive and advective heat transport mechanisms. This approach enables us to capture the complex interactions between heat diffusion and groundwater-induced thermal advection, providing a more realistic representation of thermal dynamics in heterogeneous aquifers. The MFLS model, combines features of the FLS and MILS models. It is considered a heat source of finite length and incorporates advective heat transport due to groundwater flow (v_T) along with

conductive heat transport. Thermal dispersion is implicitly handled via the thermal diffusivity ($D_t = \lambda/\mathrm{Cm}$) within the advection-conduction framework, without separate dispersivity terms as in the MILSd script. The Python script calculates the temperature increase by numerically integrating a complex expression along the finite source length (H), potentially using the method of images. The core term within the integral is a solution to the advection-conduction equation for a moving point source. The implemented equation structure is:

$$\Delta T(x,y,z,t) = \frac{Q_L}{4\pi\lambda}\exp\left(\frac{v_T x}{2D_t}\right)\int_{\text{source}}\left[\text{function\_of}(\text{erfc}, r', D_t, t, v_T)\right]\mathrm{d}z' \quad (4)$$

where $r' = \sqrt{x^2 + y^2 + (z - z')^2}$ is the distance to an element of the source. The 'function of (erfc)' part in the integrand, as seen in the script, involves terms derived from:

$$u_1 = \frac{r'^2}{4D_t t} \qquad u_2 = \frac{v_T^2 r'^2}{16D_t^2} \quad (5)$$

and combinations such as $\text{erfc}(\sqrt{u_1} \pm \sqrt{u_2}/\sqrt{u_1})$, characteristic of advective-conductive solutions. The script uses a numerically stable approach to combine terms like $\exp(\pm 2\sqrt{u_2})\text{erfc}(\ldots)$.

For all models, Q_L is the heat input per unit length of the heater, λ is the thermal conductivity, C_m is the volumetric heat capacity of the saturated porous medium, is time, and (x, y, z) are the coordinates of the observation point relative to the heat source. List of all used parameters in equations is in (Tab. 1).

2.2. Parameter estimation and model fitting

To determine the optimal thermal parameters (λ and C_m) for each analytical model, an automatic fitting procedure employing a grid search algorithm was utilized, as implemented in the Python scripts. This involved defining a range of plausible values for λ and C_m. For each pair of parameters in the grid, the corresponding analytical model was used to predict temperature changes at the observation well locations and times. The discrepancy between these model predictions and the measured temperature data was quantified using the Root Mean Square Error (RMSE). The set of parameters (λ, C_m) that minimized the RMSE was considered the best fit for that particular analytical model. The results of this grid search, including the error surface and the best-fit parameters, were visualized to assess parameter sensitivity and model performance. The following methodologies aim to rigorously assess the thermal characteristics of the aquifer, anticipating significant heterogeneity and the potential impact of groundwater flow. The forthcoming results section will elucidate the extent to which these factors influence thermal behaviour as observed through our comprehensive field tests.

3. RESULTS

This section details the outcomes of field investigations and subsequent data analyses aimed at characterizing the hydraulic and thermal properties of the heterogeneous alluvial aquifer at the Hronsek site.

Tab. 1. List of parameters used in equations.

Parameter	Name of the Parameter	Unit
Δ_T	Temperature change	°C or K
λ	Thermal conductivity	$\mathrm{W.m^{-1}.K^{-1}}$
Cm	Volumetric heat capacity	$\mathrm{J.m^{-3}.K^{-1}}$
D_t	Thermal diffusivity	$\mathrm{m^2.s^{-1}}$
H	Finite length (of heat source/heater)	M
Q_L	Heat release rate per unit length	$\mathrm{W.m^{-1}}$
r	Radial distance	m
x, y, z	Coordinates of observation point relative to heat source	m
t	Time	s
z'	Position along the source length for integration	m
v_T	Groundwater velocity	$\mathrm{m.s^{-1}}$
a_L	Longitudinal dispersivity	m
a_T	Transverse dispersivity	m
D_x	Dispersion coefficient in x-direction	$\mathrm{m^2.s^{-1}}$
D_y	Dispersion coefficient in y-direction	$\mathrm{m^2.s^{-1}}$
r'	Distance to an element dz' of the source	m
u_1	Intermediate term in MFLS equation	
u_2	Intermediate term in MFLS equation	

3.1. Site hydrogeology and hydraulic characterization

The study site encompasses a network of wells, including a central well (well 3) designated for heating, and several observation wells (3A, 3B, 3C, 3D, and the more recently installed 4A, 4B, 4C), strategically positioned to monitor subsurface responses (Fig. 2). Initial site assessment confirmed the general groundwater flow direction (as depicted in Fig. 2) and established a natural hydraulic gradient of approximately 0.01 $m.m^{-1}$.
A 3-day pumping test, followed by a 3-day recovery period, was conducted in March 2017 to assess bulk hydraulic properties. Although the recovery phase was partially affected by intermittent pumping from an unrelated nearby well, the drawdown data provided valuable insights (Fig. 5).

Analysis of drawdown data from two distinct observation wells using semi-logarithmic $t.r^{-2}$ plots (where 't' is time and 'r' is the radial distance from the pumping well) did not yield a single, consolidated curve. Instead, two clearly separated curves emerged (Fig. 6).

This divergence is a robust indicator of significant subsurface heterogeneity, suggesting non-uniform hydraulic conductivity or storativity distribution within the aquifer volume influenced by the test. Although direct thermal parameters are not derived from this hydraulic test plot itself, the principle of using t/r^2 for diagnostic purposes is analogous to its use in thermal tests.

3.2. Thermal response tests (TRTs)

A series of TRTs with varying methodologies and durations were performed to investigate the thermal transport characteristics of the aquifer.

- **Initial and exploratory heating experiments (2018, 2022):**

Early TRT campaigns included a 2-day heating test in July 2018 (2kW coil in well 3, Fig. 7A), intermittent heating cycles in August 2022 (four 1-day heating periods with 1-day breaks, 2kW, Fig. 7B), and a continuous 5-day heating test in October 2022 (2kW) (Fig. 7C).

These experiments generally produced thermal responses of limited magnitude in the observation wells, making quantitative evaluation and parameter estimation difficult (as illustrated in Fig. 7A-C). The small thermal radius of influence observed underscored the need to modify the test design, such as increased power input, longer test duration, or observation points located closer to the heat source, to achieve a more discernible thermal signal.

Extended conduction-dominated TRT (November 2023):

A more comprehensive 15-day heating test was executed in November 2023, em+ploying an electrical heater with a 2.5 kW power input in well 3. Given the saturated thickness of 5

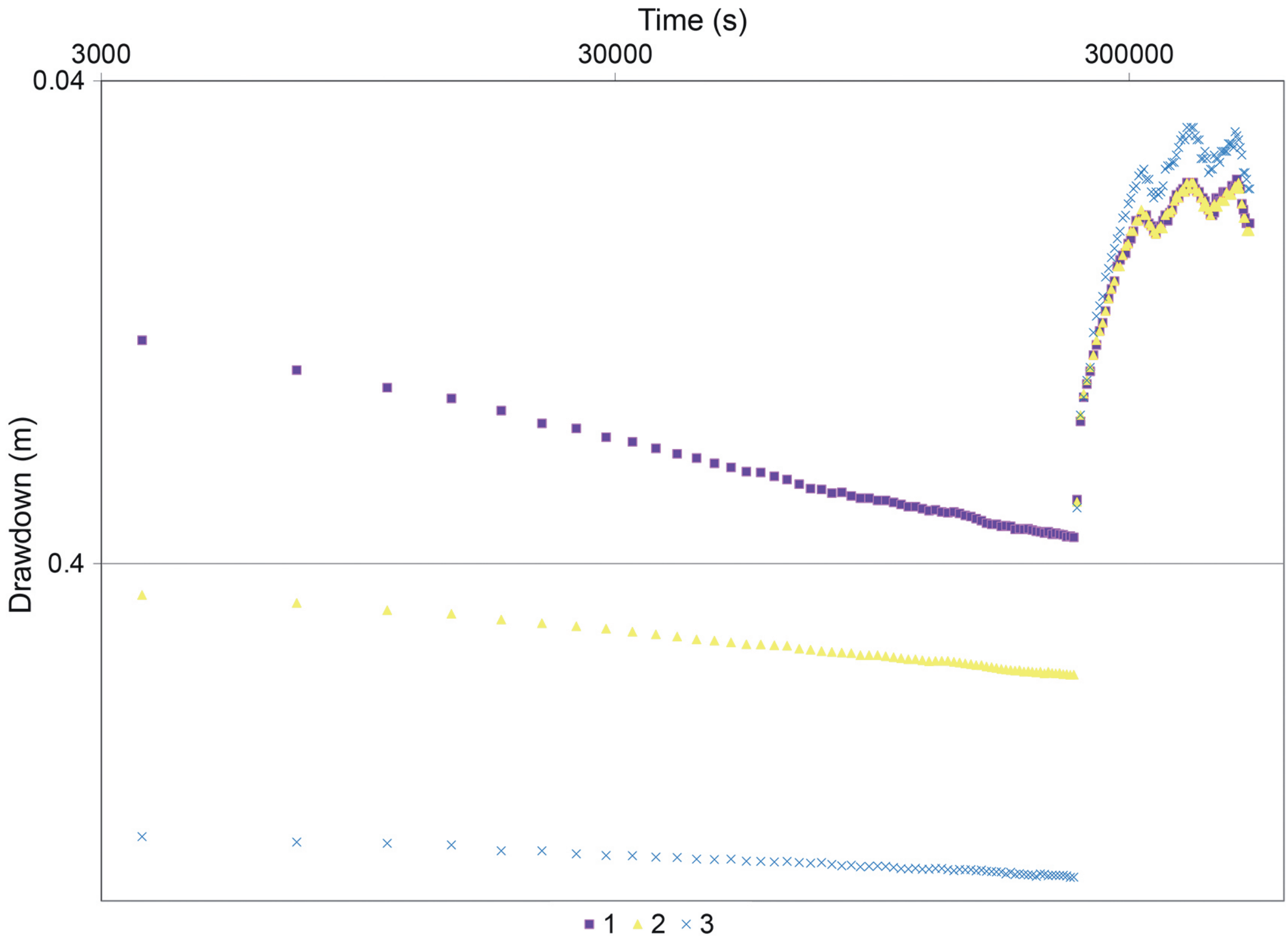


Fig. 5: Pumping and recovery test on observation wells 1, 2 and 3.

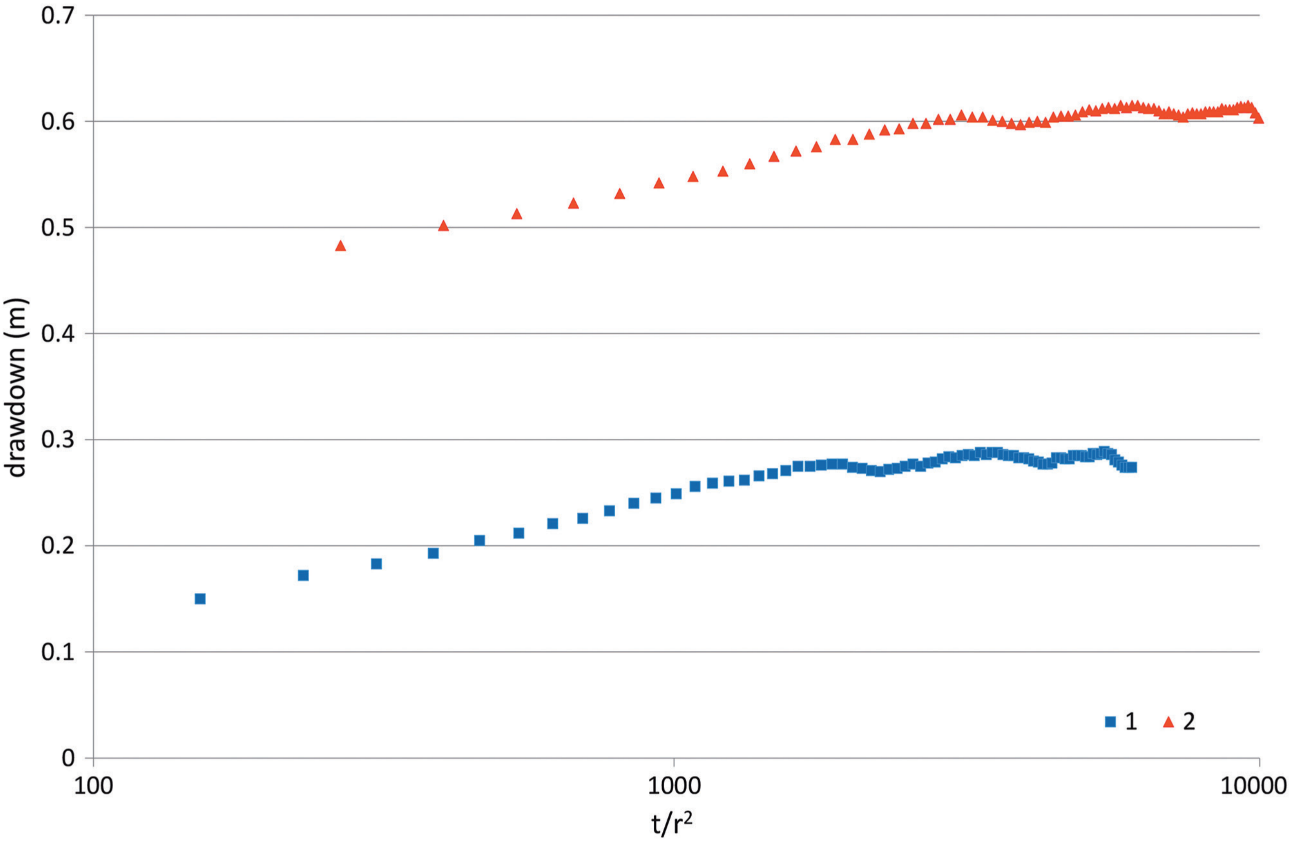


Fig. 6: Semi log chart t.r^{-2} versa drawdown on observation wells 1 and 2 shows heterogeneity in aquifer.

metres in the well, this corresponded to an approximate linear heat injection rate of 500 W.m^{-1}. For this test new observation wells (4A, 4B, 4C) were installed at closer proximity to well 3. This revised experimental setup resulted in a significantly stronger and clearly detectable thermal signal, not only in the heated well 3 but also in the surrounding observation wells (Fig. 8).

Diagnostic analysis of the temperature change (ΔT) versus t.r^{-2} from two observation wells (Fig. 9) during this extended heating test revealed distinct, non-overlapping curves.

This observation further corroborated the presence of significant thermal heterogeneity within the aquifer, consistent with the findings from the hydraulic pumping test. Fitting attempts for data segments that might correspond to influences primarily near well 3B could suggest values around $C_m \approx 2.0 \times 10^6$ J.m^3.K^{-1} and $\lambda \approx$ 6.6 W.m^{-1}.K^{-1}, while data more representative of conditions near well 4B could indicate $C_m \approx 3.9 \times 10^6$ J.m^3.K^{-1} and $\lambda \approx 7.3$ W.m^{-1}.K^{-1} (Fig. 9). The spatio-temporal evolution of the thermal plume, visualised through temperature profiles over time (Fig. 10), indicated anisotropic heat propagation. This anisotropy was influenced by the radial distance from the heat source and, to a lesser but still discernible extent, even in this primarily conduction-focused test, by the direction of natural groundwater flow.

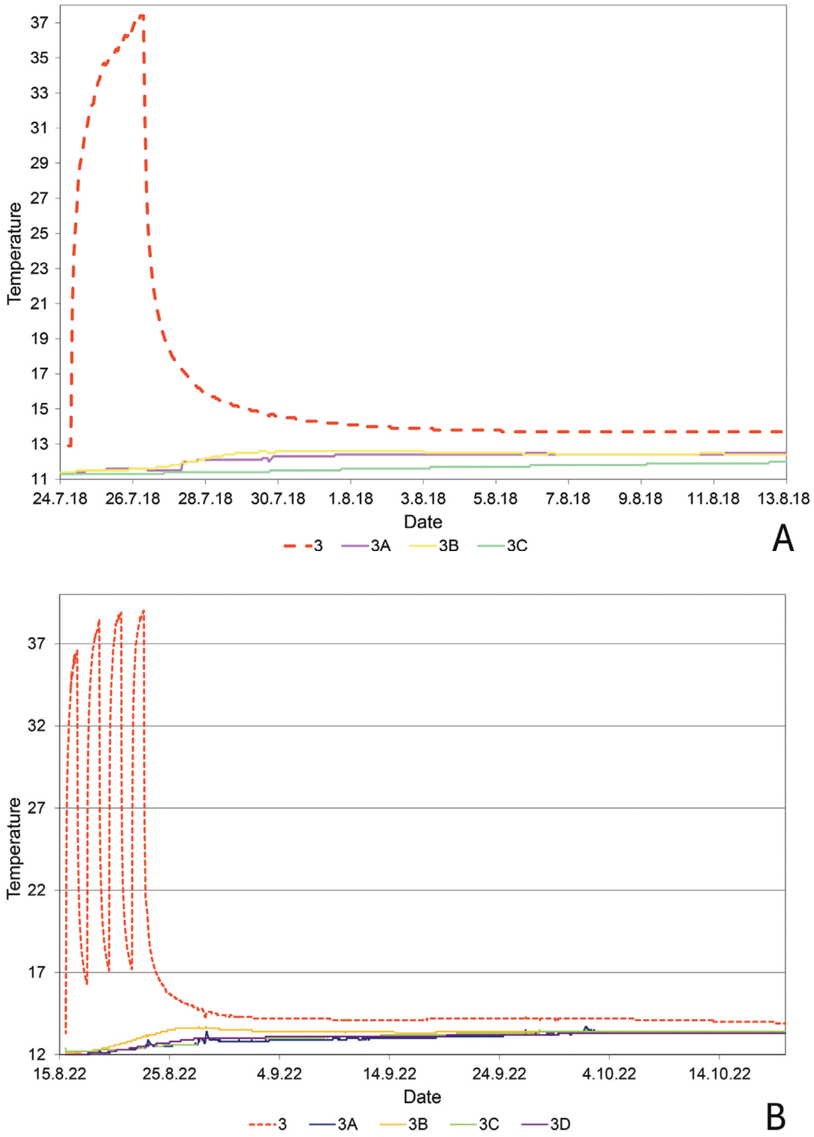
Advection-Enhanced TRT (August 2018):

An experimental test specifically designed to investigate and enhance the advective component of heat transport was conducted in August 2018. This involved the injection of pre-heated water directly into well 3 over a 4-hour period. Preliminary qualitative observations from this test (Fig. 11) suggested a significantly larger thermal radius of influence compared to purely conduction-dominated tests performed with similar energy inputs. This indicated the significant potential of advection to distribute heat more widely and rapidly. However, due to incomplete data acquisition for some parameters and logistical complexities encountered during the experiment, a full quantitative evaluation of this advection-enhanced test was not completed within the project scope.

3.3. Analytical modelling and thermal parameter estimation

The comprehensive dataset obtained from the November 2023 conduction-dominated TRT was subjected to analysis using a suite of established analytical solutions. These included the Infinite Line Source (ILS), Infinite Cylindrical Source (ICS), Finite Line Source (FLS), Moving Infinite Line Source (MILS), Moving Infinite Line Source with dispersion (MILSd), and Moving Finite Line Source (MFLS) models, as catalogued by (Stauffer et al., 2014). Primary thermal parameters, bulk thermal conductivity (λ) and volumetric heat capacity (C_m), were estimated by employing an automated grid search algorithm. This algorithm systematically sought the combination of parameters that minimised the root mean square error (RMSE) between the temperature changes measured in the observation wells and those calculated by each analytical model.

The specific best-fit parameters obtained for each model were



(Fig. 12 A-F):

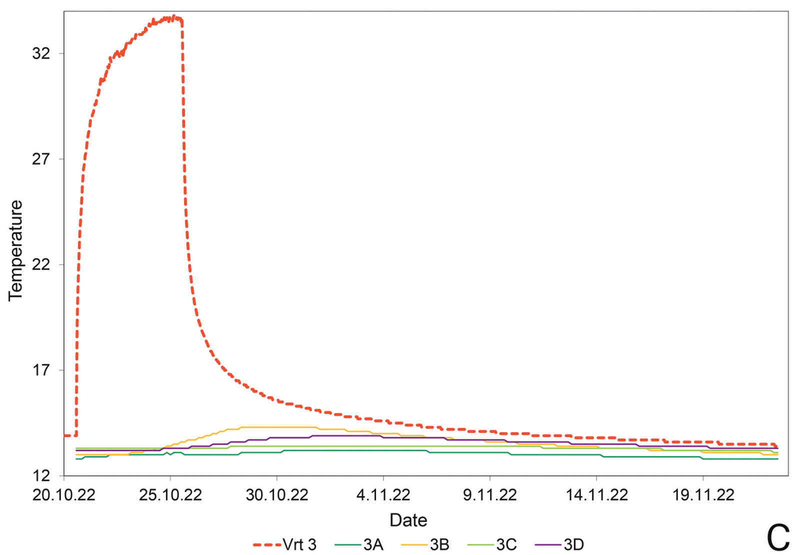


Fig. 7: A) 2-day heating test. B) 4 intermittent heating cycles. C) 5-day heating test.

Finite line source (FLS) Model:
$\lambda = 8.3$ W.m^{-1}.K^{-1} and $C_m = 2.3 \times 10^6$ J.m^3.K^{-1} (RMSE = 61.4°C). (Fig. 12 A-B).

Moving infinite line source with dispersion (MILSd) model:
$\lambda = 1.63$ W.m^1.K^1 and $C_m = 3.4 \times 10^7$ J.m^{-3}.K^{-1} (RMSE = 9.4°C). (Fig. 12 C-D).

Moving finite line source (MFLS) model:
$\lambda = 9.4$ W.m^{-1}.K^{-1} and $C_m = 7.0 \times 10^6$ J.m^{-3}.K^{-1} (RMSE = 5.4°C). (Fig. 12 E-F).

FAlthough the FLS model provides initial insights into the thermal properties assuming conductive heat transport, the incorporation of groundwater flow into the MFLS model offers a more nuanced understanding of the thermal dynamics. This enhanced approach directly addresses the focus of our study on heterogeneity and advective influences within the aquifer. Among the models tested, the MFLS model provided the best overall fit to the observed field data based on the lowest RMSE value. Visualisation of the grid search error surfaces for λ and C_m (Fig. 12 A-C) revealed that for several models, different combinations of these two parameters could produce similarly low RMSE values. This observation points towards a degree of parameter non-uniqueness and highlights the sensitivity of the estimated parameters to the chosen model structure and the inherent complexities of the subsurface not captured by the analytical solutions.

4. DISCUSSION

The results obtained from field investigations and subsequent analyses at the Hronsek site provide valuable information on the challenges and appropriate methodologies for characterising thermal parameters in heterogeneous alluvial aquifers influenced by natural groundwater flow.

4.1. Interpretation of key findings

The consistent observation of heterogeneity, evidenced by the distinct t.r^{-2} curves of the hydraulic pumping tests (Fig. 6) and the thermal response tests (Fig. 9), is a prominent feature of this study. Such heterogeneity is widely recognised as a characteristic trait of alluvial aquifers, which typically result from complex fluvial depositional processes leading to variations in grain size, sorting, and consequently,

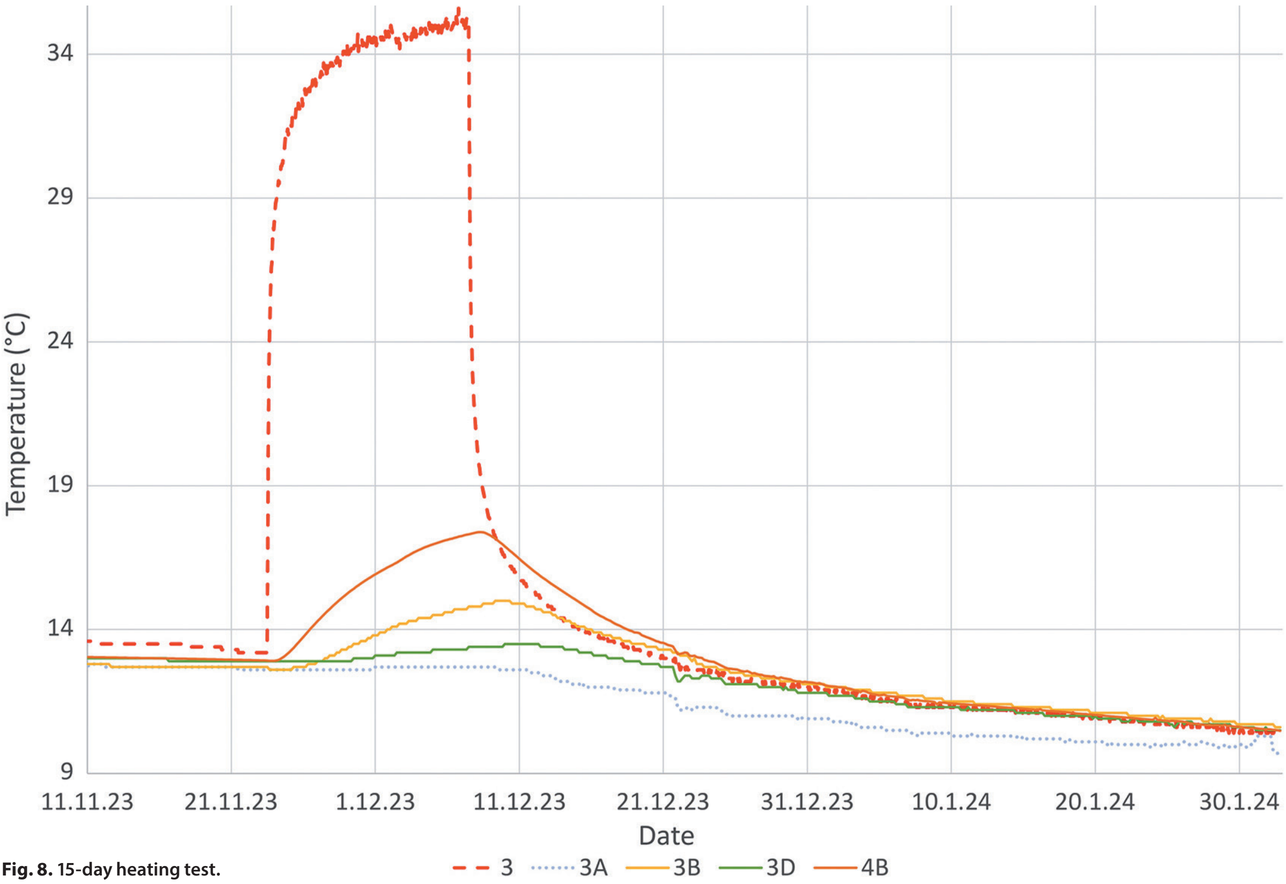


Fig. 8. 15-day heating test.

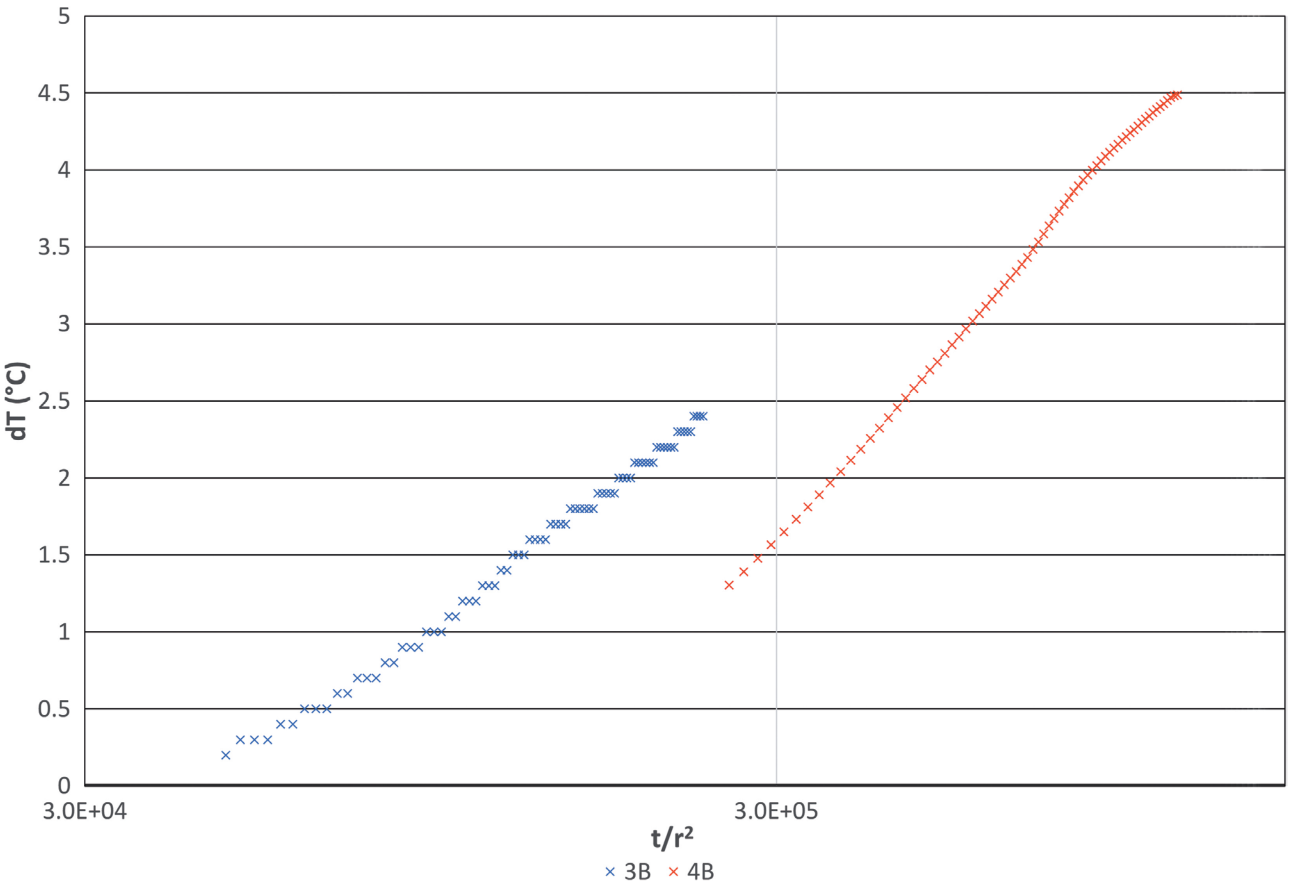


Fig. 9. The semi log chart t.r[2] versa dT shows heterogeneity in the aquifer.

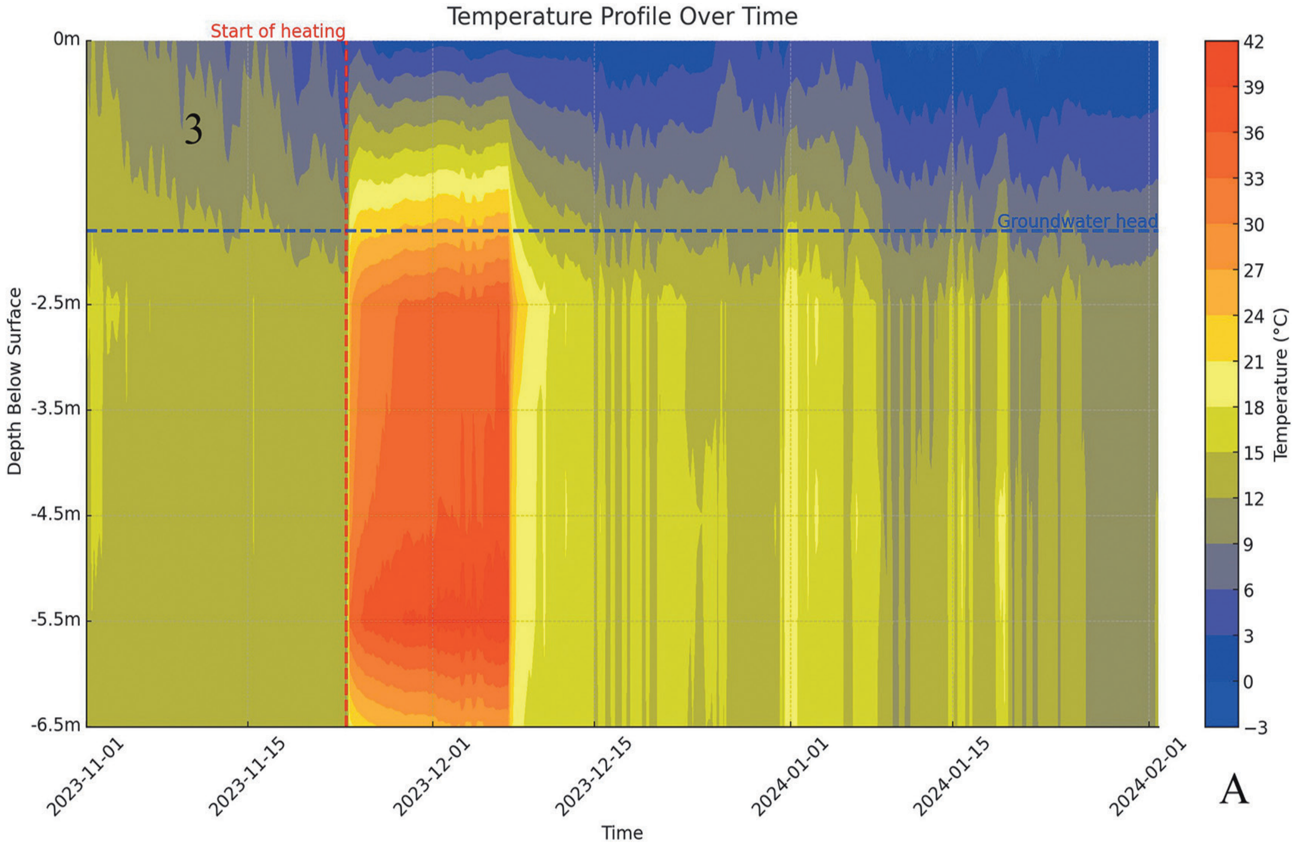


Fig. 10A: The spread of the heat wave through time (heather was in well 3).

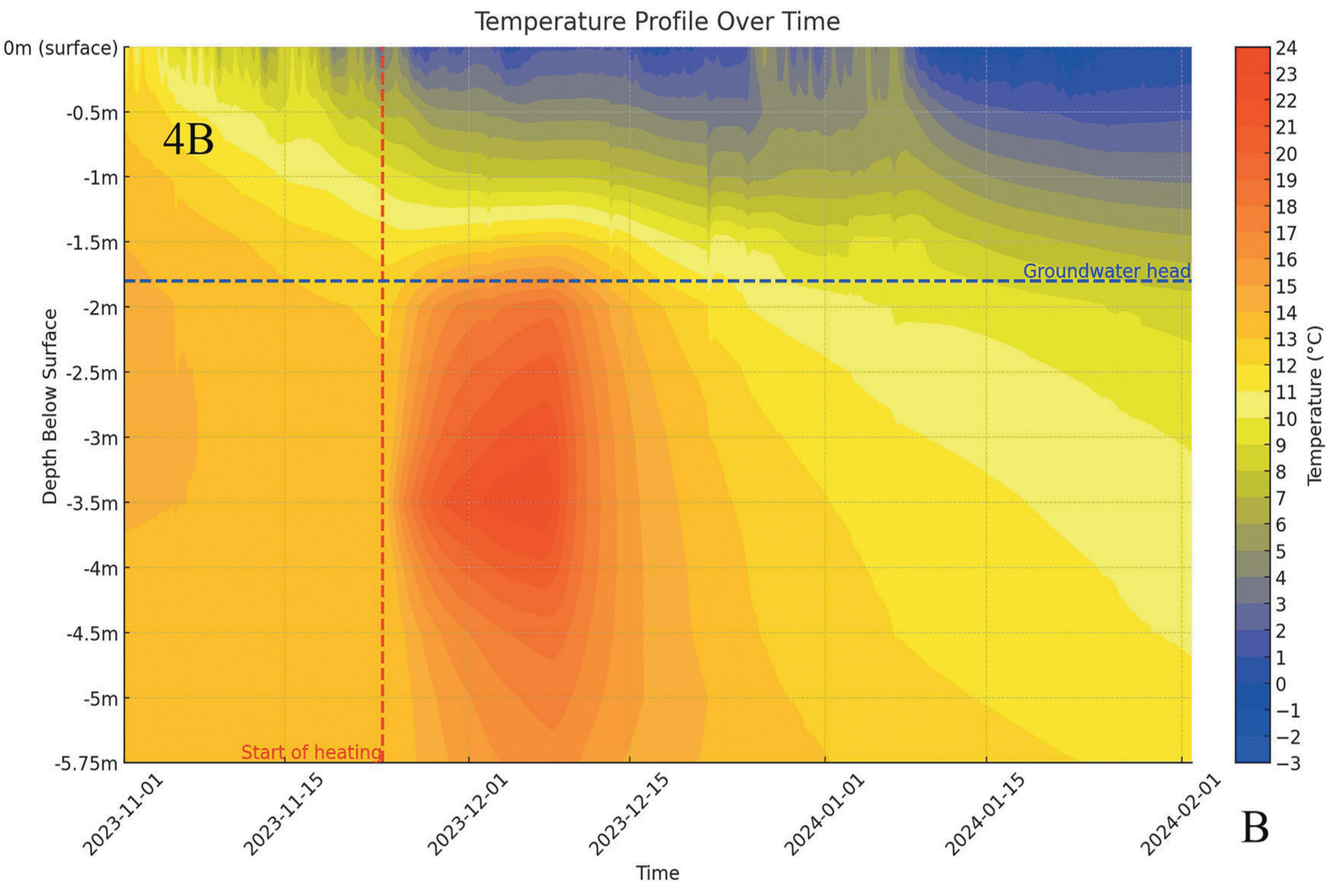


Fig. 10B: The spread of the heat wave through time (heather was in well 3).

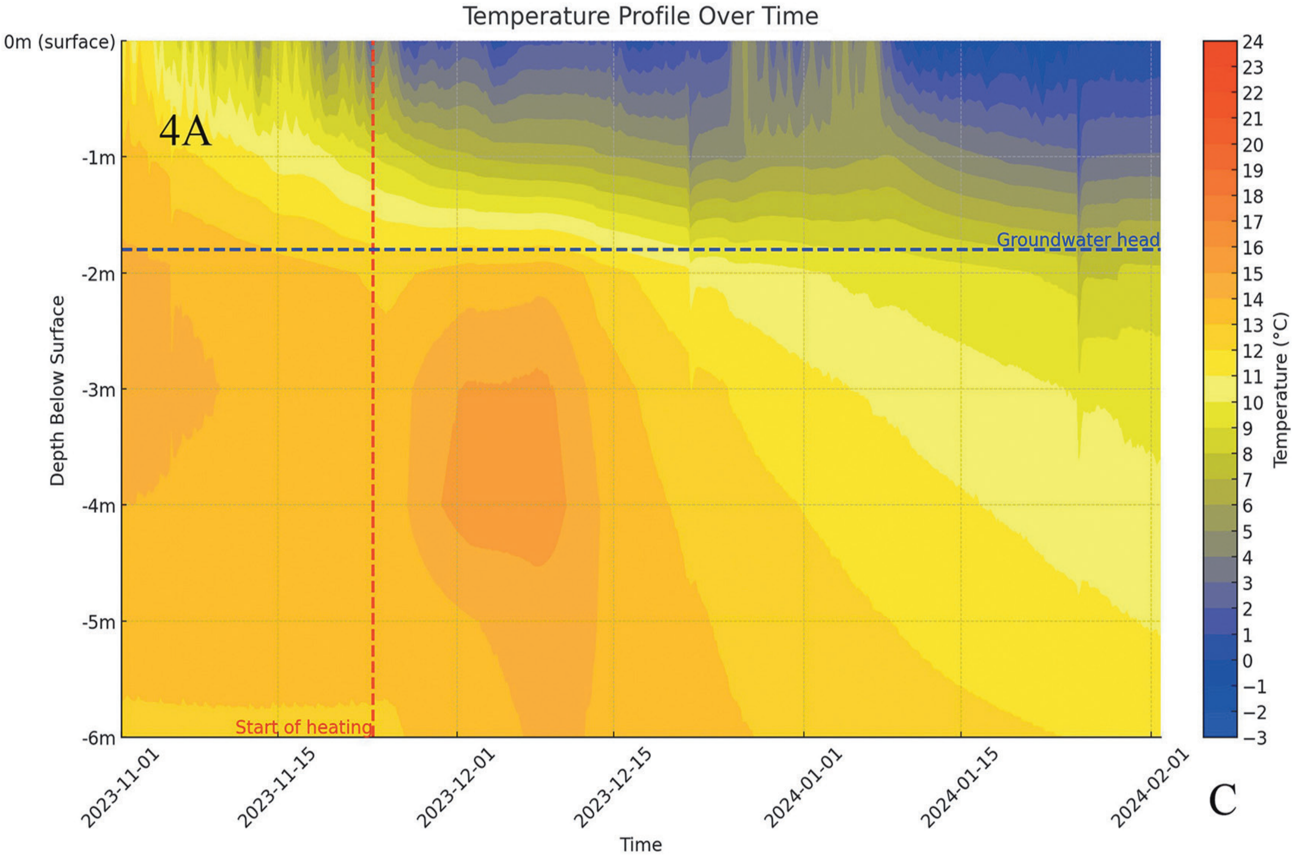


Fig. 10C: The spread of the heat wave through time (heather was in well 3).

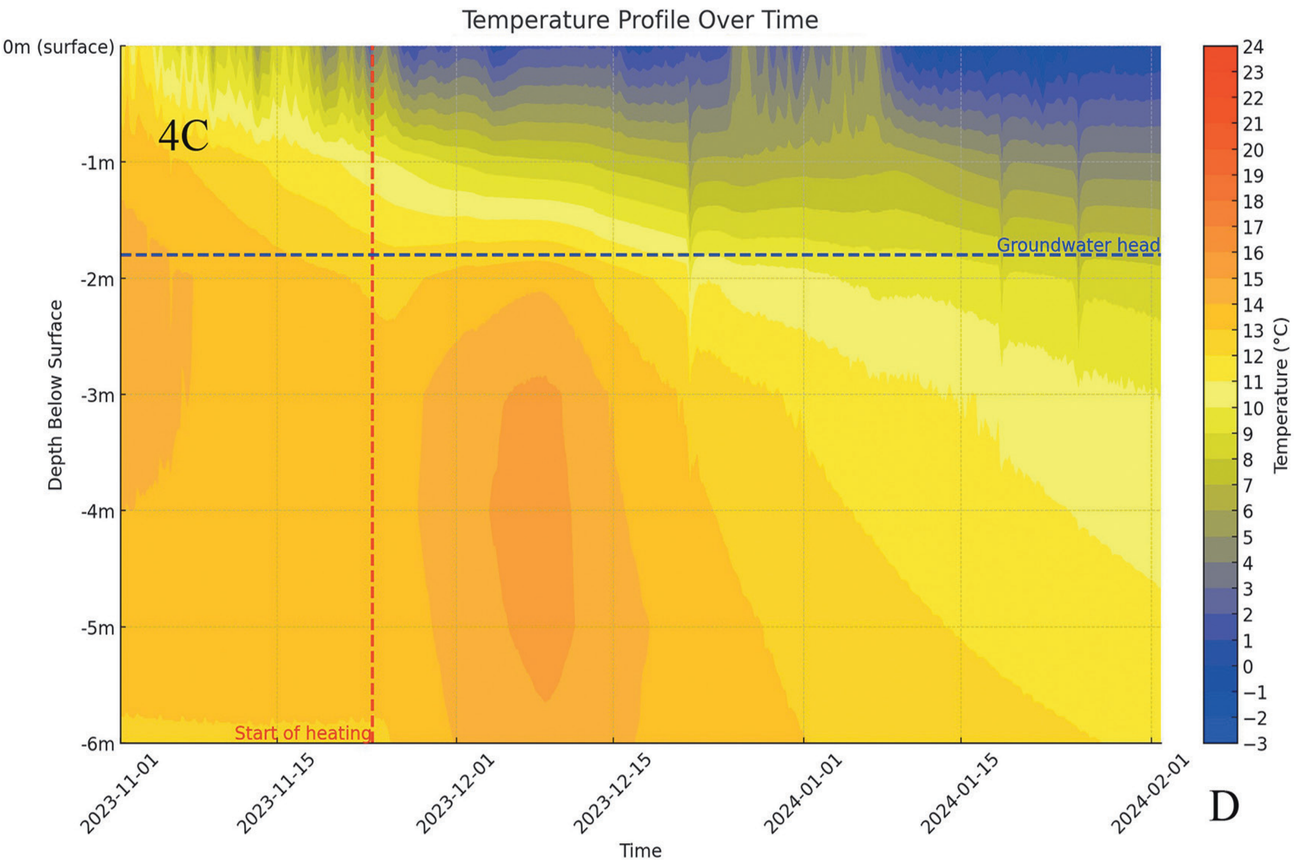


Fig. 10D: The spread of the heat wave through time (heather was in well 3).

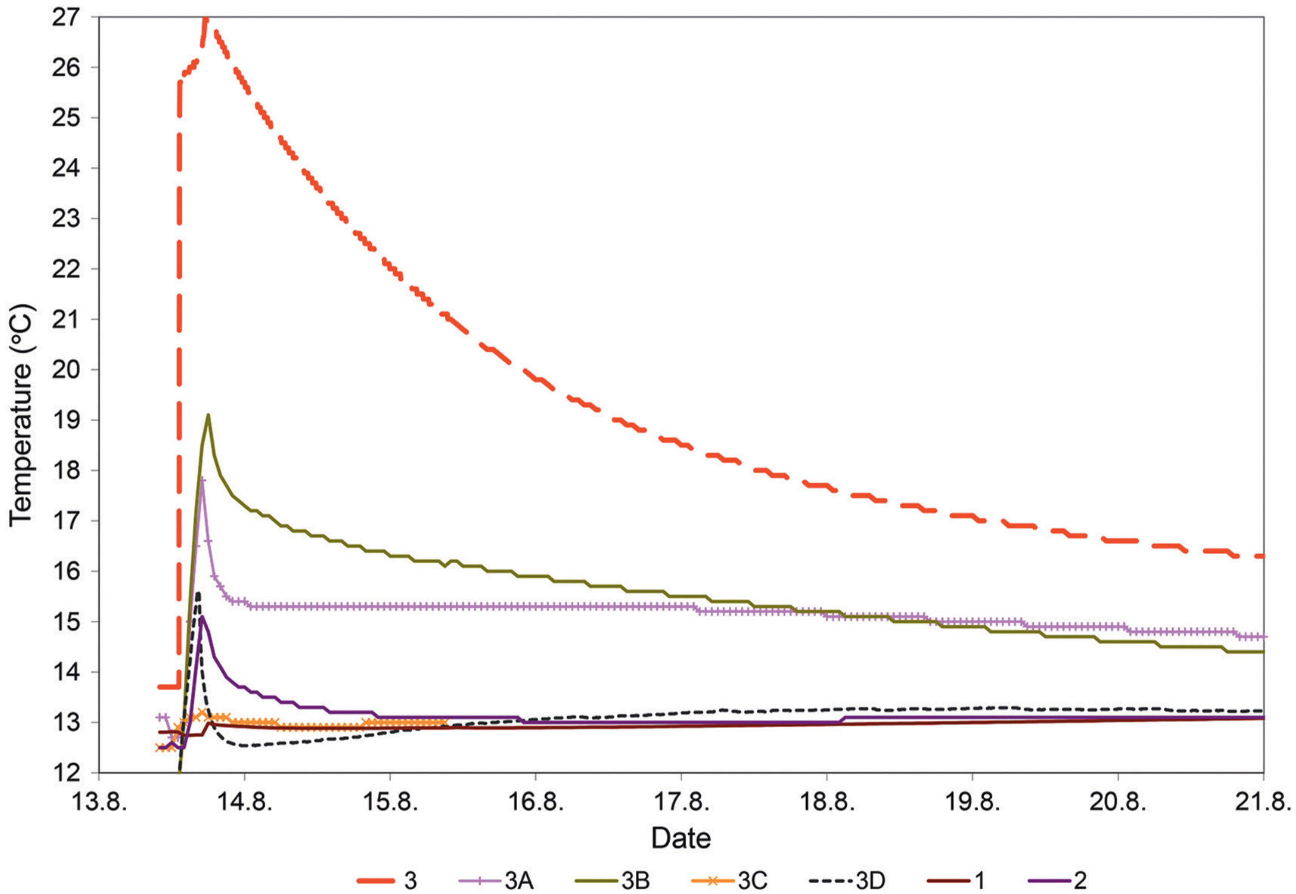


Fig. 11: Injection of preheated water directly into well 3.

hydraulic and thermal properties (Freeze & Cherry, 1979; Fetter, 2001). The diagnostic t/r^2 plots, while standard tools for identifying deviations from homogeneous behaviour (Kruseman & de Ridder, 2000; Gehlin, 2002), in this case, highlighted a degree of complexity that poses a direct challenge to the application of simpler analytical models.

The range of thermal parameters obtained from the fitting different analytical models to the TRT data is significant. The FLS model, which assumes purely conductive heat transfer from a finite line source, produced a thermal conductivity (λ) of 8.3 $W.m^{-1}.K^{-1}$ and a volumetric heat capacity (C_m) of 2.3 x 10^6 $J.m^{-3}.K^{-1}$. Interestingly, these values show some resemblance to the illustrative parameters ($\lambda \approx 6.6$ $W.m^{-1}.K^{-1}$, $C_m \approx 2.0 \times 10^6$ $J.m^3.K^1$) conceptually associated with data segments primarily influenced by conditions closer to the heat source (well 3) in the interpretation of the diagnostic t/r^2 plot from the pumping test. This similarity might arise because the FLS model, being simpler and focused on conduction without explicit advection terms, could preferentially fit early-time data or data from closer observation wells where conductive effects might dominate or where the bulk properties are more aligned with those immediately surrounding the heat source. The $t.r^{-2}$ diagnostic plot from heating tests similarly reflects evolving influences as the thermal pulse propagates, with early data potentially being more representative of near-source, conduction-dominated regimes.

On the contrary, the MILSd model, which accounts for advection and dispersion along an infinite line, produced a significantly lower thermal conductivity (λ = 1.63 $W.m^{-1}.K^{-1}$) and a significantly higher volumetric heat capacity (C_m = 3.4 x 10^7 $J.m^{-3}.K^{-1}$). The MFLS model, providing the best fit (RMSE = 5.4°C), estimated λ = 9.4 $W.m^{-1}.K^{-1}$ and C_m = 7.0 x 10^6 $J.m^{-3}.K^{-1}$. The superior performance of the MFLS model can be attributed to its ability to incorporate both the finite length of the heat source and the influence of advective heat transport, which are critical for accurately representing thermal transport in alluvial systems (Stauffer et al., 2014; Wagner et al., 2014). The substantial differences in parameters derived from MILSd versus FLS/MFLS highlight the strong influence of model conceptualisation, particularly how advection and source geometry are handled. The assumption of an infinite line source in the MILSd model and its specific formulation for advection / diffusion could lead it to interpret the data differently, possibly attributing more energy storage to the medium (higher C_m) and less efficient conductive transfer (lower λ) when trying to reconcile observed temperature changes with its inherent advective component.

The high thermal conductivity values obtained from the FLS (8.3 $W.m^{-1}.K^{-1}$) and MFLS (9.4 $W.m^{-1}.K^{-1}$) models are at the upper end or even exceed the typical literature values for saturated sandy gravels (which usually range from 2 to 5 $W.m^{-1}.K^{-1}$), e.g. Stauffer et al. (2014); Banks (2012). This discrepancy could be due to several factors:

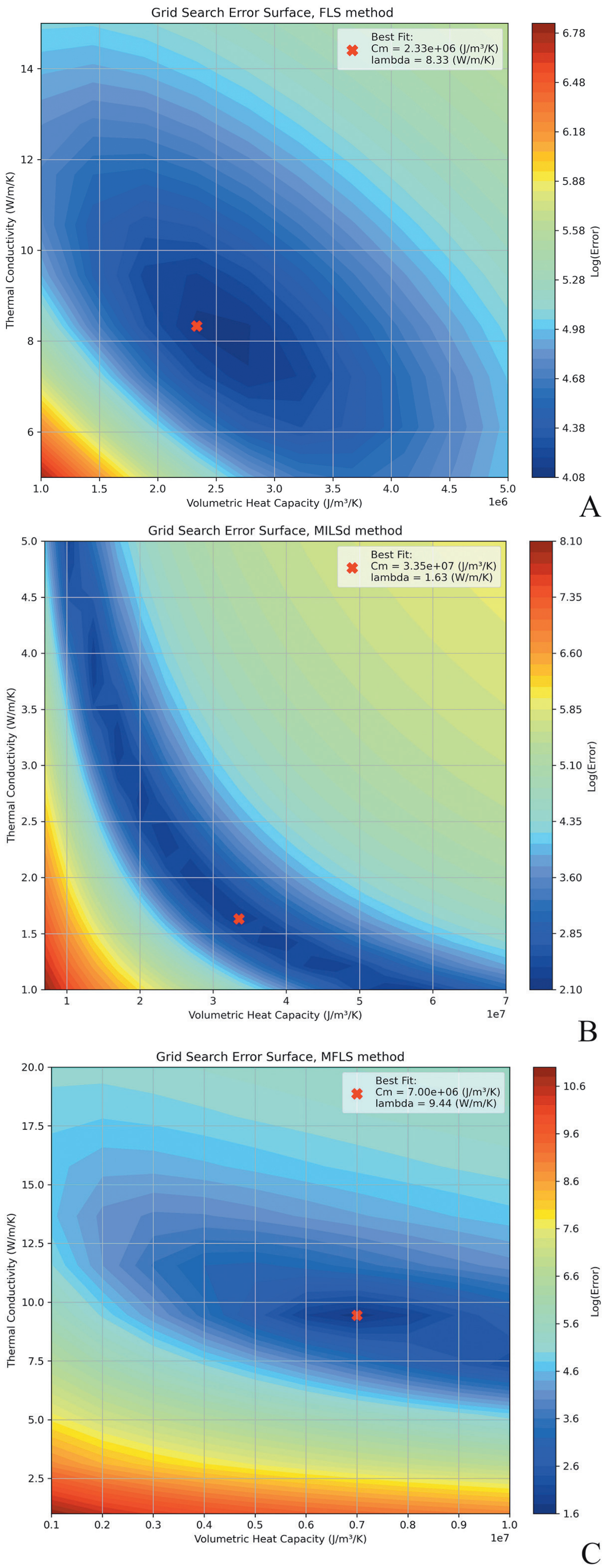
- **Enhanced convective effects not fully captured:** Even in a "conduction-dominated" test, some advective or convective heat transfer (e.g., density-driven flow within the well or near-wellbore, or influence of regional groundwater flow not perfectly accounted for by the advection term in MFLS) might be misinterpreted by the models as higher effective thermal conductivity.

- **Model limitations in heterogeneous media:** Analytical models assume homogeneity. In a heterogeneous system, preferential heat flow paths or layers with higher intrinsic conductivity could disproportionately influence the bulk parameters derived.
- **Parameter correlation/non-uniqueness:** As observed in the error surfaces of the grid search, there may be correlations between λ and C_m. The fitting process might converge to a local minimum that pairs a high λ with a specific C_m to match the observed data, even if other combinations are physically plausible. The parameter non-uniqueness observed in our grid search analysis not only highlights the complexities inherent in thermal modelling of heterogeneous aquifers but also directly correlates with the discrepancies noted between model predictions and our field data, particularly in terms of thermal conductivity estimates. This emphasises the challenge of relying solely on analytical models and underscores the need for more comprehensive modelling approaches.
- **Influence of wellbore/heater construction:** The actual heat transfer from the heater to the formation can be complex and may not perfectly match the idealised line source assumption, potentially biasing parameter estimates.

The volumetric heat capacities (C_m between 2.3 x 10^6 and 7.0 x 10^6 J.m^{-3}.K^{-1}, excluding the MILSd outlier) are generally within the expected range for saturated alluvial sediments (typically 2.0 x 10^6 to 3.5 x 10^6 J.m^{-3}.K^{-1}) e.g. Banks (2012), although the MFLS value is somewhat high. The exceptionally high C_m from MILSd (3.4 x 10^7 J.m^3.K^1) seems less physically plausible for this type of material and likely reflects the model's attempt to compensate for other discrepancies when fitting the advective component.

The divergence in parameters obtained from different models underscores the importance of careful model selection based on site conditions and test design. The fact that even the best-fitting analytical model (MFLS) still produced a notable RMSE (5.4°C) suggests that the inherent assumptions of homogeneity and simplified flow paths within these models do not fully capture the intricate reality of the Hronsek aquifer. This aligns with a broader body of research indicating the limitations of analytical solutions when subsurface conditions significantly deviate from idealised assumptions, often necessitating numerical approaches for more accurate site-specific characterisation e.g. Anderson et al. (2015); Banks (2012).

The grid search error surfaces, revealing that different combinations of thermal conductivity (λ) and volumetric heat capacity (C_m) could produce similar RMSE values, point to the issue of parameter equifinality or no uniqueness. This is a common challenge in inverse modelling, particularly when interpreting field data that may contain noise or when the conceptual model is a simplification of a more complex natural system, e.g. Beven (2006); Hill & Tiedeman (2007). It underscores the importance of careful model selection, robust parameter estimation techniques, and, where possible, incorporating independent



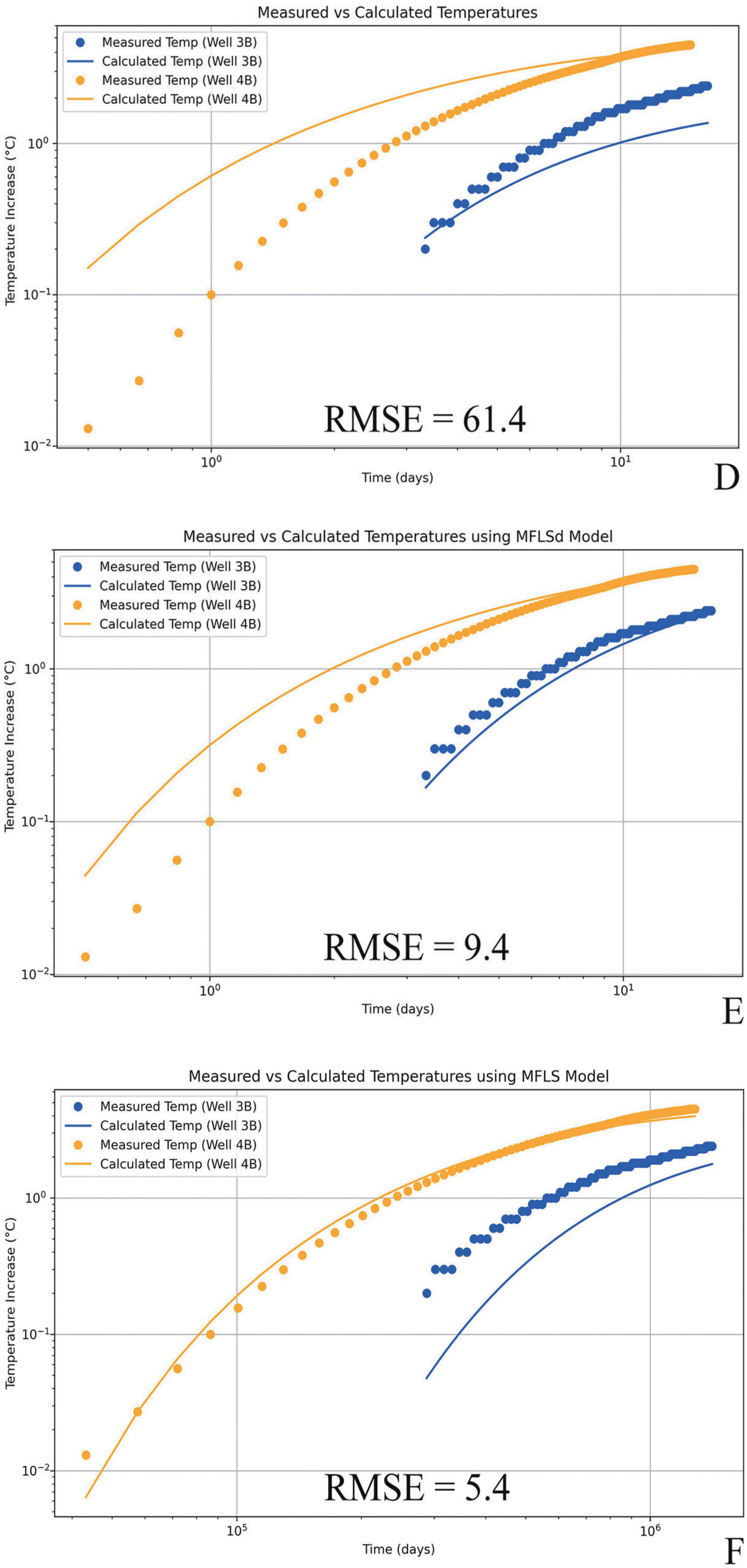


Fig. 12: **Data fitting using different analytical solutions. A) FLS grid search. B) MILSd grid search. C) MFLS grid search. D) FLS analytical solution. E) MILSd analytical solution. F) MFLS analytical solution.**

hydrogeological information or constraints to narrow down the plausible parameter space.

The preliminary qualitative results from the advection-enhanced TRT, which suggested a significantly larger thermal radius of influence, strongly support the theoretical importance of advective heat transport. In aquifers with substantial groundwater flow, advection often dominates over conduction, fundamentally altering thermal plume development and the interpretation of TRT data, e.g. Vandenbohede et al. (2011); Lamarche (2023). Although this test was not fully quantified, it serves as a practical demonstration of the potential impact of advection at the site.

4.2. Significance of findings and implications

The findings of this study have several important implications. Firstly, they highlight that for reliable thermal characterisation of heterogeneous alluvial aquifers with active groundwater flow, a multifaceted experimental and analytical approach is crucial. Relying solely on standard TRT protocols interpreted with overly simplistic analytical models can lead to considerable uncertainties and potentially erroneous estimations of thermal parameters.

Second, these uncertainties have direct practical consequences for the design, efficiency, and sustainability of subsurface thermal energy applications, such as ground source heat pump (GSHP) systems. Overestimation or underestimation of thermal conductivity and heat capacity can lead to suboptimal performance of the GSHP system (e.g. undersized or oversized borehole fields) or inefficient thermal energy storage (Florides & Kalogirou, 2007; Signorelli et al., 2007).

Third, this research contributes to the understanding of how-to better approach site investigations in similar complex hydrogeological settings. The use of diagnostic plots (t/r^2) proved effective in qualitatively identifying heterogeneity early in the evaluation process. The iterative approach to TRT design, learning from initial tests to optimise later ones (e.g., by adding closer observation wells and extending test duration), is also a valuable lesson.

4.3. Limitations of the study

While this study provides significant information, certain limitations should be acknowledged. Advection-enhanced TRT, although promising, could not be fully quantitatively evaluated due to data gaps and logistical challenges, preventing a direct comparison of parameters derived under different flow regimes. The primary interpretation relied on analytical models, which, as discussed, have inherent simplifications. A more detailed geological characterization of the site, perhaps through additional borehole logging or geophysical surveys, could further refine the understanding of subsurface heterogeneity and help constrain numerical models.

4.4. Conclusions and recommendations for future work

In conclusion, this study successfully characterized key aspects of thermal transport in a heterogeneous alluvial aquifer, demonstrating the challenges posed by natural variability and groundwater flow when using standard field techniques and analytical models. The MFLS model offered the best analytical approximation (λ = 9.4 W.m^{-1}.K^{-1}, C_m = 7.0 x 10^6 J.m^{-3}.K^{-1}) of the system's thermal behaviour among those tested, though the estimated thermal conductivity appears high compared to typical literature values. The variability in parameters derived from different analytical models and the diagnostic evidence of heterogeneity highlight the limitations of these approaches in complex geological settings.

Based on these findings, the following recommendations are made:

1. **Numerical modeling:** Develop a 3D numerical groundwater flow and heat transport model for the Hronsek site. This model should incorporate the observed heterogeneity (if possible, based on available geological data or inferred from hydraulic/thermal tests) and the natural hydraulic gradient to simulate the TRT experiments more realistically.
2. **Further investigation of advection-dominated tests:** If feasible, repeat or redesign the enhanced TRT with robust data acquisition protocols to allow quantitative parameter estimation under conditions where advection is more dominant. This would provide valuable data for validating numerical models under different transport regimes.
3. **Anisotropic thermal properties:** Future modelling efforts should consider the potential for anisotropic thermal conductivity, which is common in sedimentary formations and can significantly influence the geometry of the heat plume.
4. **Integrated parameter estimation:** Explore integrated approaches for parameter estimation, possibly combining data from both hydraulic and thermal tests within a unified inverse modelling framework using numerical models.

By addressing these recommendations, a more complete and accurate understanding of the thermal dynamics of the Hronsek groundwater system can be achieved, ultimately supporting a more effective and sustainable use of subsurface thermal resources.

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