Modal analysis of subsoil using Fourier transformation under dynamic loading

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Abstract: When designing geotechnical constructions, it is important to know its behavior under dynamic loading, which is induced or natural seismicity. It is known from the solution of dynamically loaded structures that if the excited frequency of the load is equal to, or approximately equal to, the natural frequency, undesirable resonance phenomena occur. A reliable tool for investigating the dynamic properties of geotechnical engineering is the modal analysis, which uses its stiffness and mass properties to find the natural frequencies. The original "discrete" Fourier transformation (DFT) of the time function into the frequency domain found practical use only in 1965, when the American mathematicians James William Cooley and John Tukey substantially made it more efficient. It is thus possible to compare the solution of the differential equation of the natural oscillation of the tunnel lining structure with the experimental analysis based on the measurements, their subsequent processing and evaluation using the fast Fourier transformation (FFT). The article discusses the measurement of technical seismicity during real tunnel excavation and its use to control its dynamic effects on nearby structures. It also evaluates the effects of technical seismicity in different geological conditions and their connection with natural seismicity, which represents a major input to the design of geotechnical structures in seismically active areas.

Keywords: Fourier transformation, induced seismicity, dynamic loading, earthquake, natural frequency

1. INTRODUCTION

Every designer's task is to create a reliable structure. An important starting point for designing and assessing underground geotechnical structures is knowledge, which progresses through two stages. The first stage involves conservative thinking based on empirical knowledge; the second stage involves an intuitive grasp of the interrelationships and dependencies of the investigated entity in analytical and numerical form (Yang et al., 2025). Responsible designers who create functioning designs have no choice but to describe their acquired knowledge mathematically, along with all the associated problems, necessary logic, and level of complexity of the solution. This is usually determined by the calculation model's requirements (Duan et al., 2024; Godlewski et al., 2023; Jasiński and Grzyb, 2022). Such a model, within a certain range of validity, can generally be understood as an object, either real or virtual, that exhibits a certain degree of similarity in behaviour to the modelled object. The model's primary function is to analyse the object without causing damage, whether for economic or ethical reasons. Two extreme cases can be distinguished between which the model can move. In the first case, the model explains only the basic principle of the phenomenon under investigation; in the second case, the model is a simulation that does not require any numerically relevant description of reality.

Determining the load on a construction is one of the most fundamental and challenging geotechnical tasks (Ortuta and Tóth, 2019). Unlike the tasks faced by static engineers in earthworks and structures, where external loads are relatively precisely defined, the role of geotechnical engineers in underground construction is far more challenging (Marence, 2003; Tabatabaei Mirhosseini, 2017; Szajna et al., 2024). Dynamic shocks, whether of natural or technical origin, frequently occur during the operation of underground structures. One way to numerically describe a seismic event is to measure and record the acceleration over time (An and Liu, 2019). In areas with a low incidence of earthquakes, estimated inputs based on long-term observations and probability calculations are used in the calculations. During tunnelling, it is possible to measure the environment's response to explosive rock loosening and thus also the environment's seismic response to the generated energy (Chabroňová et al., 2017). However, these measurements may be waived if the mining authority decides otherwise, in which case it is up to the contractor whether they will be carried out.

Dynamic shocks related to tunnelling are a type of induced anthropogenic seismicity. This type of seismicity is caused by human activity (An et al., 2021) and is characterised by smaller tremors than those caused by earthquakes. This seismic activity changes the stress in the Earth's crust, and a key feature is its low magnitude (An et al., 2021) and is characterised by smaller tremors than those caused by earthquakes. The seismic hazard (Pachla and Tatara, 2022) from induced seismicity can be assessed using techniques similar to those used for natural seismicity, although non-stationary waves must be considered. This approach suggests that tremors caused by human activity resemble those that cause earthquakes, although the difference in depth of origin must be considered. This means that models based on natural earthquakes, which are represented in numerous databases, can be used. A risk assessment would then be carried out, taking into account the seismic hazard and its impact on objects near the epicenter. Ultimately, the risk can be mitigated by adjusting the hazard (Deng et al., 2018).

Tunnelling affects the primary stress of the surrounding rock environment, causing deformation and seismic activity. This is associated with problems during underground excavation and, of course, poses a risk to workers. This process is known as 'rock burst', i.e. the spontaneous failure of the mechanical and stabilising properties of the material — in this case, rock — associated with seismic shocks (Bakoš et al., 2018). Many

Manuscript received 2025-06-05
Revised version accepted 2025-07-14

underground structures therefore operate seismic monitoring networks to manage the risk of artificial tremors and guide excavation procedures in the event of a threat to surrounding buildings. Measurements of the seismic effects of blasting during tunnel excavation were also used in this paper to assess them in various geological environments and their connection with natural seismicity, which is incorporated into geotechnical designs.

2. GEOLOGICAL DESCRIPTION

Measurements were taken during the excavation of three tunnels: two in Slovakia and one in the Czech Republic (Fig. 1). These tunnels were excavated within three distinct geological structures that have undergone different historical developments.

Homole Tunnel

The Homole Tunnel is a motorway tunnel on the D35 Ostrov–Vysoké Mýto section in the Czech Republic. Passing through the Vraclav Ridge, it encounters genetically varied strata of Quaternary soils, as well as bedrock, which in the investigated locality is represented by Cretaceous variable sandy claystones and siltstones, with local transitions to limestones.

The siltstones and sandstones found in the Vraclav Ridge area are marine sediments from the Jizera Formation (Middle to Upper Turonian). The highest deposits consist of streaky calcareous sandy siltstones with calcareous concretions. These form the top layer of bedrock on Homole Hill and on the eastern slopes of the Vraclavsky Ridge. Beneath these are the characteristic grey, highly friable claystones, into which a layer of mottled, calcareous, dusty-to-calcareous sandstones several metres thick is embedded, with the grey claystones continuing below (Matouš, 1972). Generally, a variably strong eluvial layer of gravelly to stony clay of varying thicknesses has developed on the bedrock surface. Its thickness is strongly influenced by past erosion processes. The structural layers of the bedrock are significantly influenced by the historical development of the site. The dominant tectonic feature is the Vraclav anticline, and the Homole tunnel will pass through its eastern edge. The Vraclav anticlinal marks the boundary between two parts of the Bohemian chalk basin: the Vysoké Mýto syncline to the east and the Chrudim chalk to the west. A continuous water table has been identified in one common aquifer in the tunnel area. The water table is free or only slightly stressed. Near-surface weathering and the loosening of the rock mass, together with abundant tectonic faulting, interconnect four otherwise hydrogeologically separate units.

Okruhliak Tunnel

The Okruhliak Tunnel forms part of the R4 expressway in the Prešov section. It passes through a rock massif formed by a complex of Neogene aleuritic-pelitic rocks — the Prešov Formation (Lower Miocene – Egenburg). In the area around the western portal, where the effects of technical seismicity were measured, and on the western slopes of the partial ridge adjacent to the tunnel corridor, layers of grey and blue-grey claystones and silty claystones prevail. In the eastern direction, grey and dark grey claystones with laminated and thin-plate stratification, typically exhibiting shale and shale-like disintegration, dominate. The massif is broken by systems of fractures inclined at 45 - 60°, 70 - 80° and occasionally 80 - 90°. These fractures are linked to the inclination of the stratification and tectonic faulting of the rocks (Martinčeková et al., 2014). From a hydrogeological perspective, due to the complex filtration inhomogeneity and low permeability of the overlying sedimentary formations, only a small proportion of precipitation and Quaternary groundwater enters the Neogene molasse formation complex. The interconnection of more permeable areas enables limited, shallow groundwater circulation.

Bikoš Tunnel

The Bikoš Tunnel is located on the R4 expressway in the Prešov section. According to the regionalization of the Western Carpathians from an engineering-geological perspective, the territory

Fig. 1: Location of measurements

belongs to the Carpathian flysch region and the flysch uplands area. Its geological structure comprises sediments from the Paleogene period of the Inner Carpathians, represented by the Hutian and Zuberec formations, as well as Neogene molasse sediments from the Prešov and Teriakov formations. In the southern part, where the tunnel is located, the Hutian formation is predominantly present, consisting of layers of claystones, clayey siltstones and sandstones. The claystones are predominantly laminated and very thinly bedded with plate-like disintegration. Sandstones are predominantly fine-grained and medium-bedded with laminar development. The massif is characterised by a layered structure with laminated to thin-plate stratification, generally arranged subhorizontally with a slope of 10 - 30°, and occasionally up to 70° (Martinčeková et al., 2014). From a hydrogeological point of view, these are low-permeable sediments and the rock environment is characterised by fracture permeability.

3. MEASUREMENT OF SEISMIC RESPONSE DURING BLASTING

Blasting tests were conducted directly during excavation work on the Homole, Okruhliak and Bikoš tunnels. The seismograph positions were chosen to capture the geological types of the material environment as accurately as possible. In this case, these were siltstones (Fig. 2), claystone (Fig. 3), and sandstone (Fig. 4). Measurements were taken continuously for 48 hours, with several blasts recorded as the coining progressed, causing the

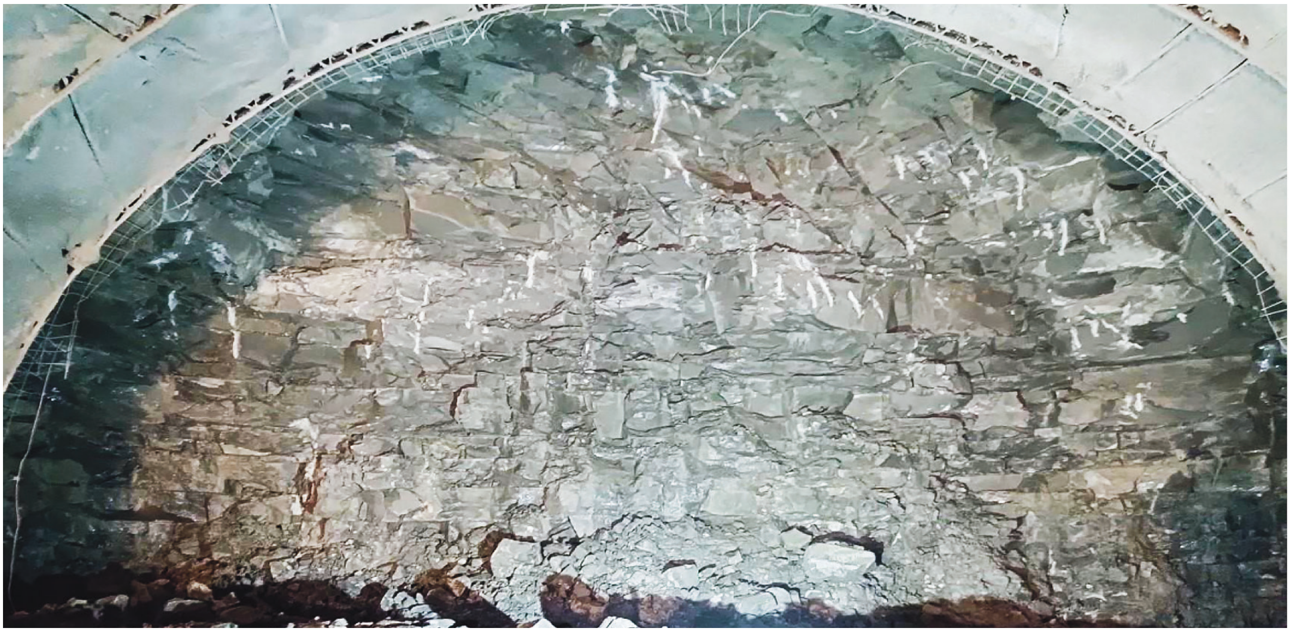

Fig. 2: Siltstones at the headwall of the Homole tunnel

Fig. 3: Claystones at the headwall of the Okruhliak tunnel

distance to the headwall and the actual measuring devices to change.

Digital seismic equipment (BRS32) from ARENAL Ltd. was used to measure and graphically record the seismic effects of blasting. The equipment was equipped with a three-component seismic sensor (SM6, Sensor Nederlanden, Netherlands). The BRS32 apparatus records waveforms in a rectangular coordinate system. Two of the axes lie in the horizontal plane (N, oriented towards the blast; E, perpendicular to the blast), and the third (Z) axis is oriented vertically to the ground. The time data on the seismic records are in UTC (Coordinated Universal Time), which is registered by the Garmin satellite time receiver connected directly to the BRS32 equipment. UTC is 2 hours ahead of SELČ (Central European Summer Time) and 1 hour ahead of CET (Central European Time). The BRS32 instruments were placed at the measuring stations and switched on for continuous automatic recording. The course of the individual N (tunnel axis), E (perpendicular tunnel axis) and Z (vertical) components of seismic waves during blasting was recorded at the measuring stations. The data recorded by the seismic equipment was automatically stored in its memory and then decoded into ASCII format for processing.

As previously mentioned, the blasts were automatically recorded

Fig. 4: Sandstones at the headwall of the Bikoš tunnel

Fig. 5: Position of seismographs at the site Homole tunnel (A: in tunnel; B: on the portal; C: on the tunnel stage; D: on the ground directly above the blast)

Fig. 6: **Position of seismographs at the site Okruhliak tunnel (A: in tunnel; B: on the portal)**

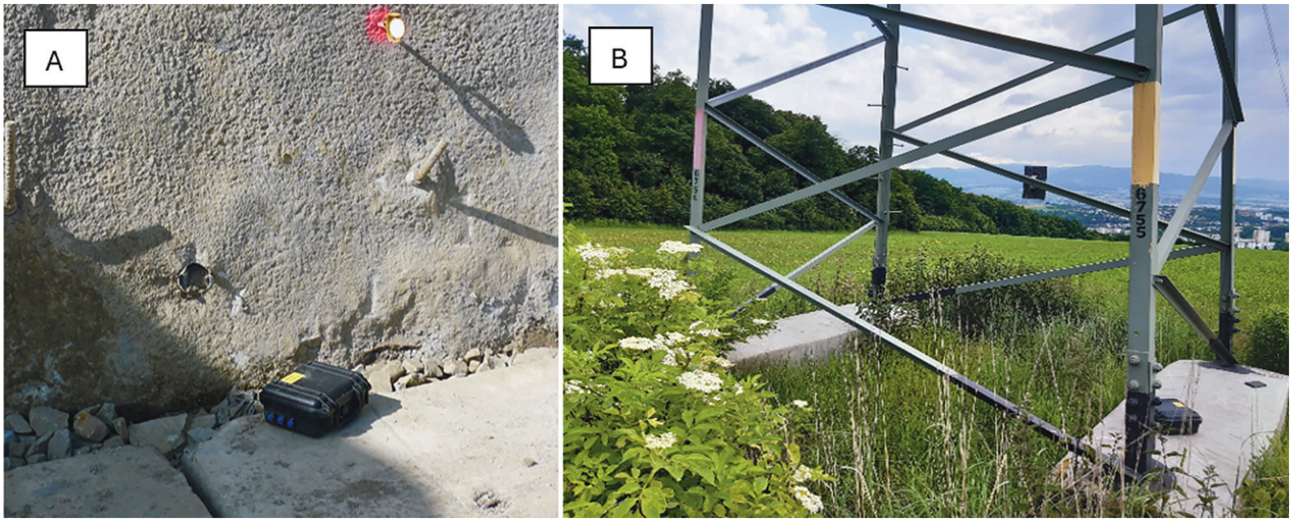

Fig. 7: **Position of seismographs at the site Bikoš tunnel (A: on the portal; B: on the concrete base of the mast directly above the blast site)**

over a period of 48 hours at various points within the tunnels. Measurements were taken directly in the tunnel to track horizontal accelerations and at the surface (possibly directly above the blast site) to record vertical accelerations. It was these surface measurements that constituted the simulation of natural seismicity (see Figures 5, 6 and 7).

4. EVALUATION OF MEASUREMENTS AND DETERMINATION OF NATURAL OSCILLATIONS

As mentioned in the introduction, when describing a seismic event numerically, it is necessary to record its speed over time. The Fourier transform is used to evaluate time records, including accelerograms, by converting them from the time domain to frequency components in order to determine their periodicity. Each continuously differentiable function with period T *(valid: $f(t)=f(t+T)$)* can be developed into a trigonometric series (Bubeník et al., 2010):

$$\hat{f} = \frac{a_0}{2} + \sum_{k=1}^{\infty}\left(a_k \cos\frac{2\pi kt}{T} + b_k \sin\frac{2\pi kt}{T}\right) \tag{1}$$

with coefficients:

$$a_k = \frac{2}{T}\int_0^T f(t)\cos\frac{2\pi kt}{T}dt,\ \mathrm{k} = 0, 1, 2, 3, \ldots \tag{2}$$

$$b_k = \frac{2}{T}\int_0^T f(t)\sin\frac{2\pi kt}{T}dt,\ \mathrm{k} = 0, 1, 2, 3, \ldots \tag{3}$$

Due to the computational complexity that arises when processing extensive time records, it is practical to consider a system of equations in a complex form and use some properties of trigonometric functions. The system of equations can be further considered in the form:

$$\{f\} = \left[W_n\right] \cdot \{\hat{f}\}, \tag{4}$$

where:

$\{\hat{f}\}$ is the time record in seconds (s),

$\{f\}$ are the unknown frequencies in hertz (Hz),

$[W_n]$ is the Vandermonde matrix of the nth order

$$[W_n] = \frac{1}{\sqrt{n^3}} \begin{bmatrix} w_n^0 & w_n^0 & w_n^0 & . & . & w_n^0 \\ w_n^0 & w_n^1 & w_n^2 & . & . & w_n^{(n-1)} \\ w_n^0 & w_n^2 & w_n^4 & . & . & w_n^{2(n-1)} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ w_n^0 & w_n^{(n-1)} & w_n^{2(n-1)} & . & . & w_n^{(n-1)(n-1)} \end{bmatrix}, \quad (5)$$

For the elements of a square matrix, which is in complex form, the following applies:

$$w_n^{jk} = \cos\left(2\pi \frac{(j-1)(k-1)}{n}\right) + \mathrm{i} \cdot \sin\left(2\pi \frac{(j-1)(k-1)}{n}\right), \quad (6)$$

where:

i is an imaginary unit: $i^2 = -1$.

The following figures (Fig. 8) represent the evaluation of the measured (vertical acceleration) values for a single blast on three differently spaced seismographs for Homole tunnel.

BRS 48; measurement directly in the rock mass at 60 tunnel meters (40 meters in front of the blast) (Fig. 5; left).

BRS 14; seismograph placed on the surface vertically above the blasting site (Fig. 6; right).

With these measurements, it was possible to proceed to the Fourier transform and evaluate the spectral velocity at 0 % attenuation (Fig. 9). It is the main input for determining the stiffness of the subsoil under dynamic loading, which enters into the calculation of the waveforms of the geotechnical structure.

As part of the evaluation of the measured data, the response of the subspace to the excited vibration was compared for different geological material environments (Fig. 10). From the measured data, the influence of the layers emplacement of the seismic wave propagation velocity is evident.

The measurements themselves showed how important it is to set up the seismographs correctly when recording the blasting-induced vibrations. Seismographs located in the tunnel record values that are directly related to the construction of the primary lining and its stiffness. For these parts of the structure, measurements are the main input to the dimensioning process and subsequent design of the secondary lining for seismic loads. Seismographs that were placed on the surface, either above the blast or on portal sections, recorded velocities and accelerations in a range of values corresponding to natural seismicity. Simply described, the vibrations did not vibrate the rigid structure but vibrated the subspace as a whole, and this corresponds to the vibration of a given lithological genotype in a classical earthquake.

Fig. 11 presents the progress of the two earthquakes in conjunction with the blast measurements. This is a seismographic record of the devastating earthquake in Armenia on 8.12.1988 and the earthquake recorded in Slovakia near the village of Ľubovník on 9.10.2023.

It is evident that the vibrations induced by the blasting itself are more intense but short-lived, which falsely suggests a preconceived notion of structural safety. However, when using the Fourier transform, fringing regions are visible that approach the 33 Hz threshold (Fig. 12).

When calculating the effects of natural seismicity, it is necessary to consider eigenforms with a frequency lower than 33 Hz and at the same time take into account those eigenforms that in total absorb at least 90 % of the released kinetic energy.

5. DETERMINATION OF THE NATURAL VIBRATION OF THE STRUCTURE

The mechanical properties of the civil structure are its boundary conditions, stiffness determined by the elasticity of the material and the thickness of the structure, and the weight associated

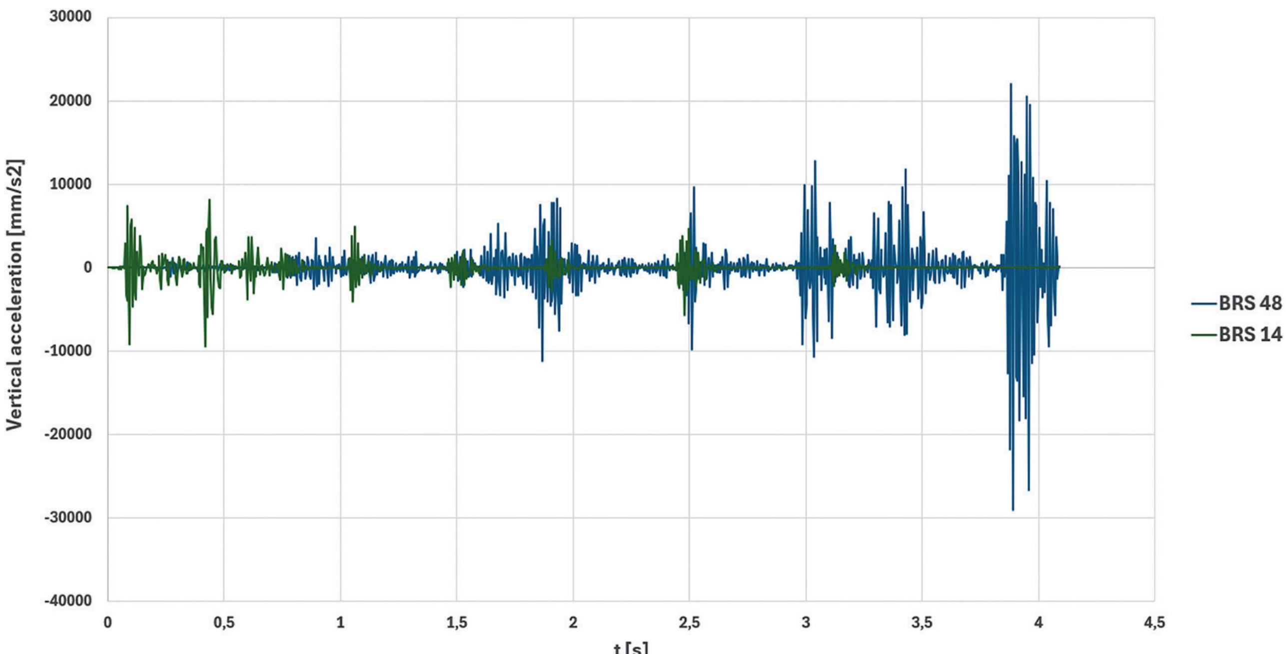

Fig. 8: Vertical acceleration (mm/s²) for Homole tunnel

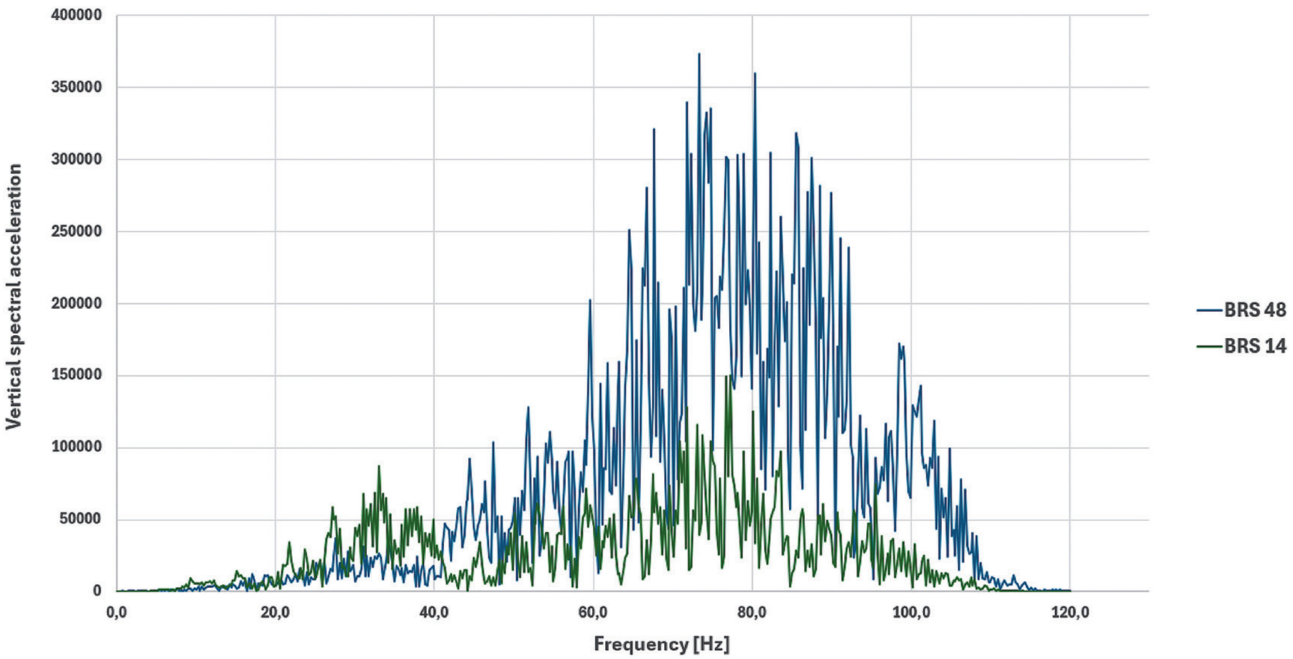

Fig. 9. Vertical spectral acceleration for Homole tunnel

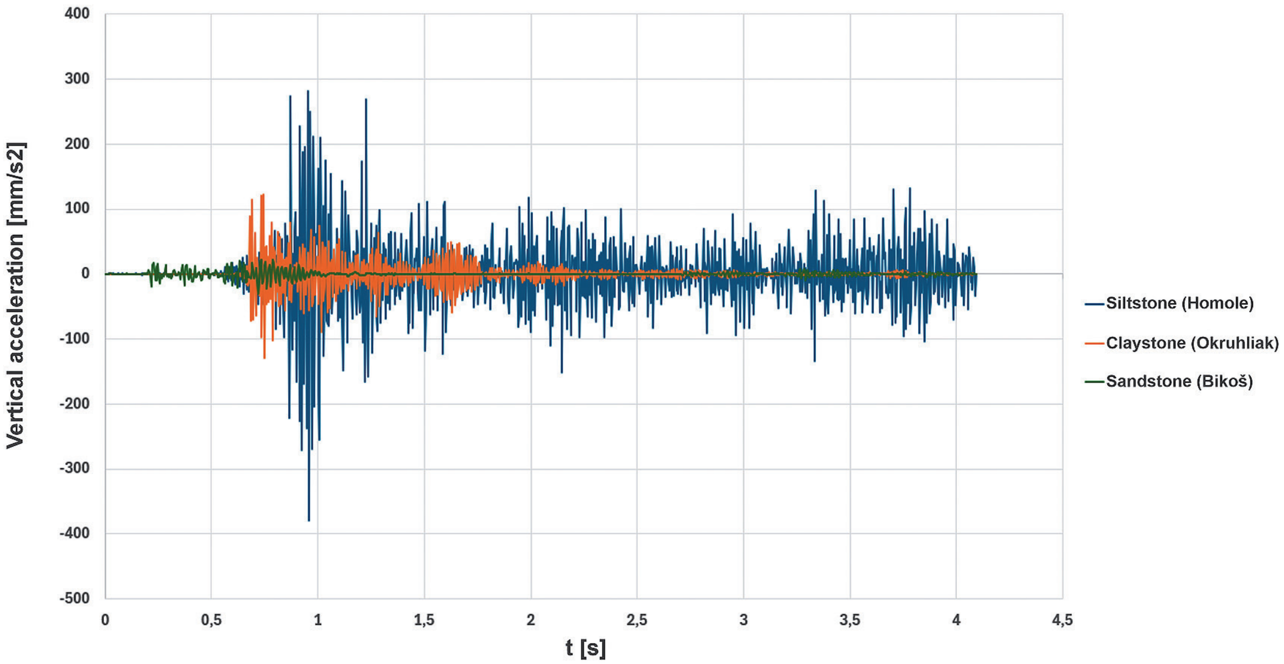

Fig. 10: **Vertical acceleration (mm/s^2) for Homole tunnel (siltstones), Okruhliak tunnel (claystones) and Bikoš tunnel (sandstones)**

with it. The quantities of self-oscillation obtained by calculation, with the help of which it is possible to understand the behavior of the structure, are natural frequencies, natural deformations and modal masses.

There are several methods that can be used to calculate the natural oscillation of a structure. In practical calculations, the subspace iteration method developed by the German civil engineer Klaus Jürgen Bathe, or the Lanczos method (Lanczos, 1950), developed by the Hungarian mathematician Cornelius Lanczos, is most often used.

Another practical item of information that is part of modal analysis is the eigenform of the oscillation. This is a dimensionless deformation that informs the designer about how the structure deforms in a particular mode of oscillation. Each eigenfrequency is assigned to its own deformed shape. The natural deformed shape of the structure is obtained from the natural frequency, the mass matrix and the stiffness matrix.

The last piece of practical information that is part of modal analysis is the "modal mass". This is also a dimensionless number and this value informs how which eigenform absorbs kinetic energy. Using the modal mass, it is possible to distinguish between significant eigenforms that already contribute to the real (measurable) deformation and, conversely, from the designer's point of view, not to deal with less significant eigenforms. The

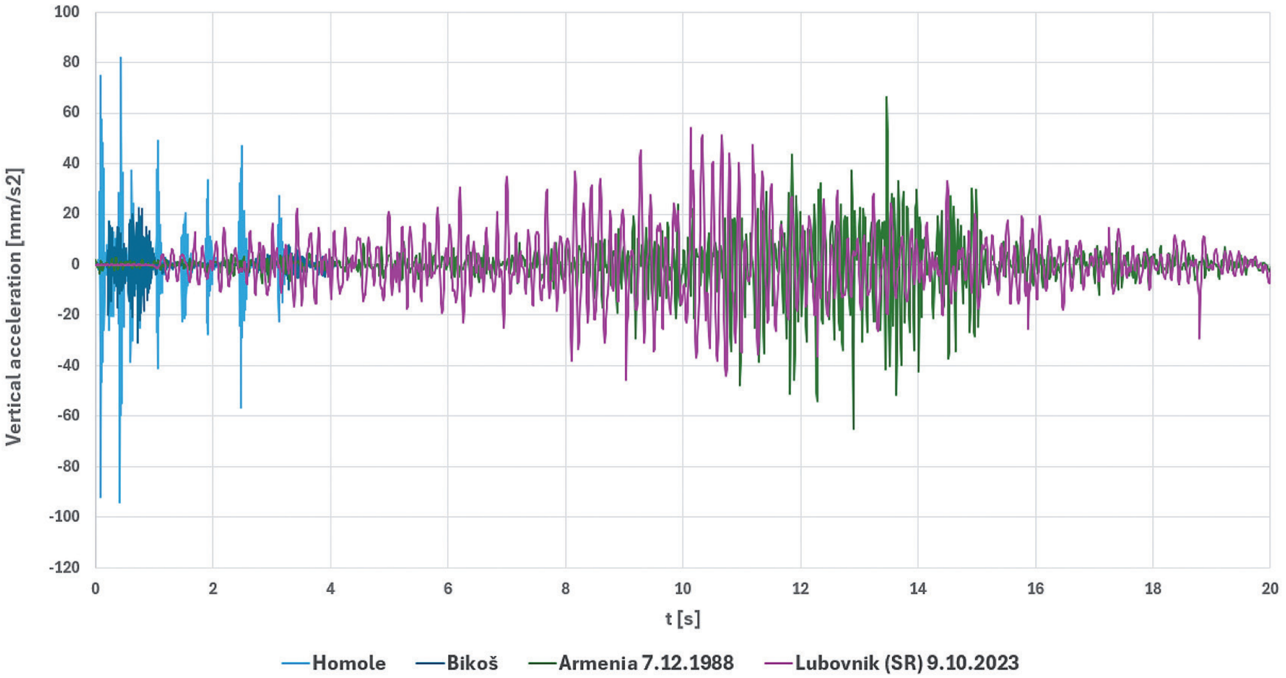

Fig. 11: Vertical acceleration for earthquake and blastings

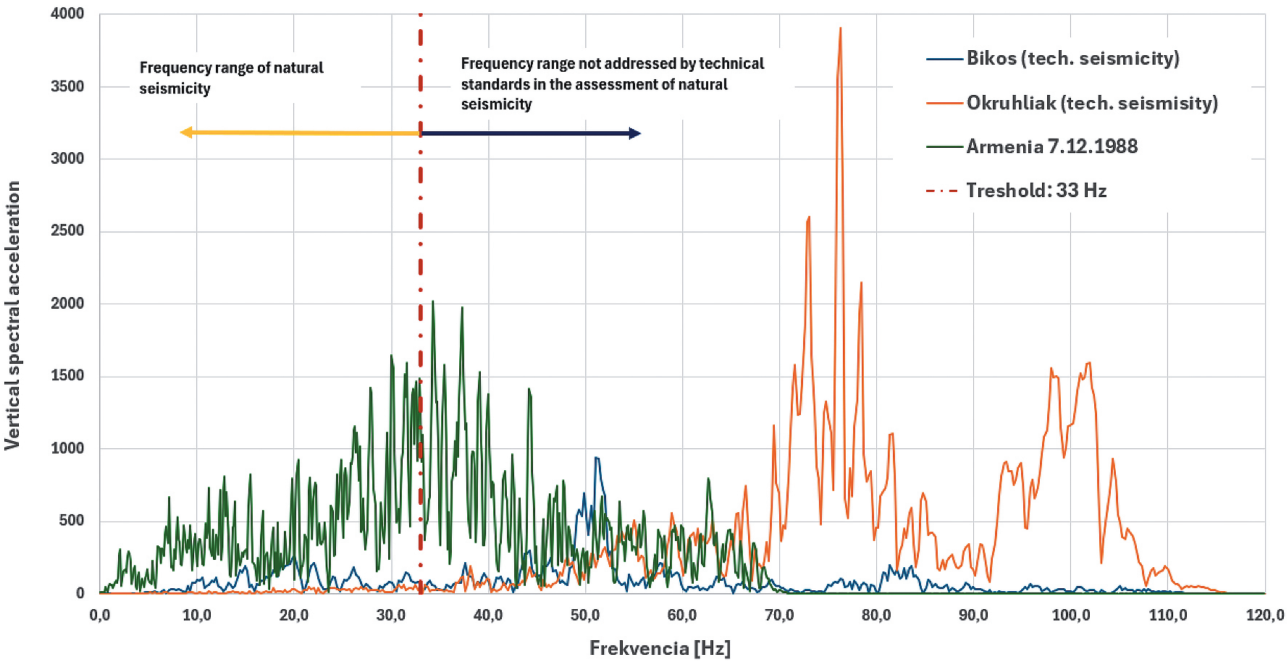

Fig. 12: Vertical spectral acceleration in Z direction for earthquake and blastings

modal mass can be quantified as a percentage. All calculated eigenmodes absorb 100 % of the kinetic energy coming from the dynamic load. The modal mass is obtained by multiplying the values of the dimensionless eigenstrains (eigenforms) and the mass quantified in kilograms.

The self-oscillation of the secondary lining of the tunnel can be described using the differential equation of a plate on an elastic foundation (Bareš, 1989):

$$d_1 \cdot \frac{\partial^4 w_z}{\partial x^4} + 2d_1 \cdot \frac{\partial^4 w_z}{\partial x^2 \partial y^2} + d_1 \cdot \frac{\partial^4 w_z}{\partial y^4} + k_z w_z + k_{xy} u_x + k_{xy} v_y - \rho \cdot \frac{\partial^2 w_z}{\partial t^2} = 0, \quad (7)$$

where:

$$d_1 = \frac{E\,h^3}{12\,(1-\mu^2)} \text{ is the plate bending stiffness,} \quad (8)$$

w_z, u_x, v_y is the solution of the differential equation,
E is the modulus of elasticity of the material,
h is the thickness of the plate,
μ is Poisson number,
ρ is specific gravity of the material,
k_{xy} is modulus of subgrade reaction for the domain of seismic deformations in shear
k_z is modulus of subgrade reaction for the domain of seismic deformations in uniform pressure.

The secondary lining of the tunnel is considered as a separate dilation unit, modeled by plate elements according to Kirchhoff's theory, which are placed on an elastic foundation. Inertia effects are approximated by a consistent mass matrix in the natural oscillation calculation.

6. BACK-ANALYSIS OF TUNNEL LINING BASED ON REAL SEISMIC LOADS FOR HOMOLE TUNNEL

Based on the data recorded by seismographs (the measuring instrument was located directly in the Homole tunnels) and subsequently evaluated and processed using the Fourier transform, it was possible to determine the eigenforms and the corresponding natural frequencies (Figs. 13 and 14). In the modal analysis, 22 eigenmodes that contribute significantly to the seismic response were investigated. Their relative modal mass is approximately 90 %. Table 1 summarizes these measurements and directional calculations. Figs. 15, 16 and 17 represent the outputs of the calculation with the visible influence of the seismic load, evaluated based on measurements of induced seismicity. Table 2 summarizes the calculated values that resulted from the assessment of the secondary lining. In a multiscale modal response, a reasonable number of eigenmodes of oscillation that contribute to the seismic response must be

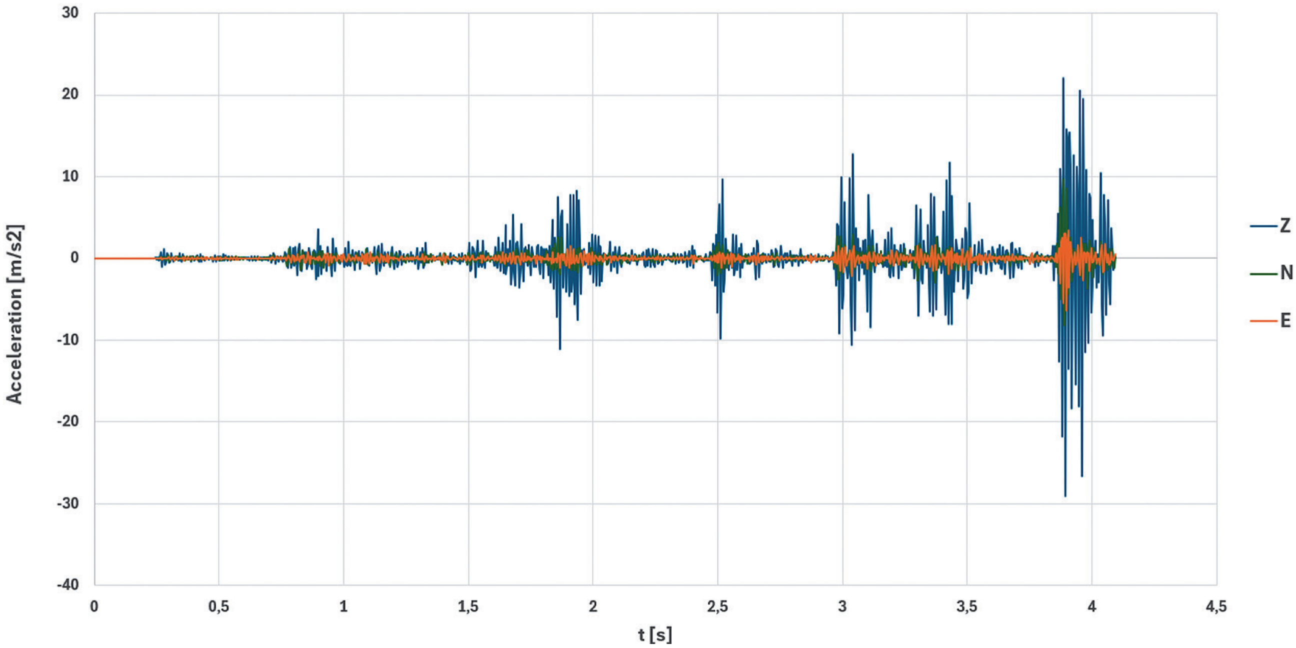

Fig. 13: Acceleration (mm/s^2) for Homole tunnel (BRS48: Fig. 4; A)

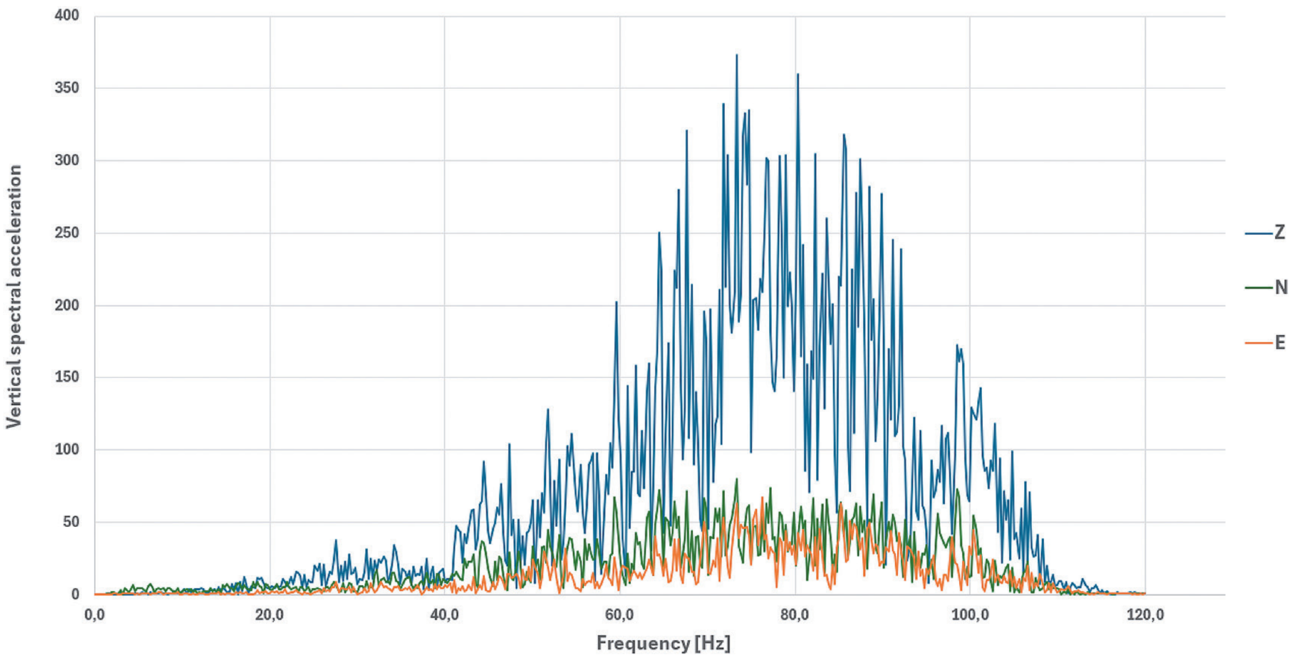

Fig. 14: Vertical spectral acceleration for Homole tunnel

Table 1: Natural frequencies calculated by the subspace iteration method

Eigenform	Angular frequency [rad/s]	Eigenfrequency [Hz]	Eigenperiod [s]
1	244,29	38,88	0,026
2	244,35	38,89	0,026
3	269,83	42,94	0,023
4	269,86	42,95	0,023
5	328,03	52,21	0,019
6	329,31	52,41	0,019
7	370,07	58,90	0,017
8	370,10	58,90	0,017
9	371,18	59,07	0,017
10	371,21	59,08	0,017
11	460,02	73,21	0,014
12	461,28	73,42	0,014
13	513,27	81,69	0,012
14	538,93	85,77	0,012
15	538,99	85,78	0,012
16	554,83	88,30	0,011
17	571,32	90,93	0,011
18	586,74	93,38	0,011
19	586,78	93,39	0,011
20	587,73	93,54	0,011
21	590,91	94,05	0,011
22	619,09	98,53	0,010

considered. (It is recommended to consider at least 90 % of the effective modal weight.) This recalculation has already been carried out with using in situ measured and analyzed seismic acceleration data. Fig. 15 and 16 show the principal stresses of the secondary lining.

7. CONCLUSION

This paper focuses on induced seismicity, its measurement and evaluation in relation to the design of geotechnical structures. When seismographs are properly deployed for blasting monitoring, natural seismicity is also evaluated. This aids the design and subsequent sizing evaluations for dynamic loads associated with earthquake resistance. These measurements, evaluations and subsequent geotechnical calculations form an iterative process in which each stage (measurement, evaluation and calculation) produces a result, but the main output is the overall evaluation. Close cooperation between geologists and geotechnical engineers is therefore essential, as the results are shared.

The article discusses mathematical methods that can be used to evaluate and describe the phenomena associated with a seismic event. A conceptual model of the tunnel lining is employed, incorporating subgrade stiffnesses that roughly align with the spectral analysis of the accelerogram measured during tunnel excavation.

When calculating the effects of natural seismicity, eigenforms with a frequency lower than 33 Hz must be considered, as well as those eigenforms that absorb at least 90 % of the released kinetic energy in total. Only the shape of the deformation can be determined from the eigenforms. The size of the deformation is determined by the amount of energy released, quantified in technical standards for natural seismicity as peak acceleration. Peak acceleration is primarily determined by the location of the seismic event, the subsoil type and the response spectrum. When calculating the inertial forces that cause stress on the structure, static loading is calculated and combined using either the SRSS (square root sum of the squares) or the CQC (complete quadratic combination) method. This "combination" of significant eigenforms is considered an extraordinary load condition in the static assessment. In the given example, inertial

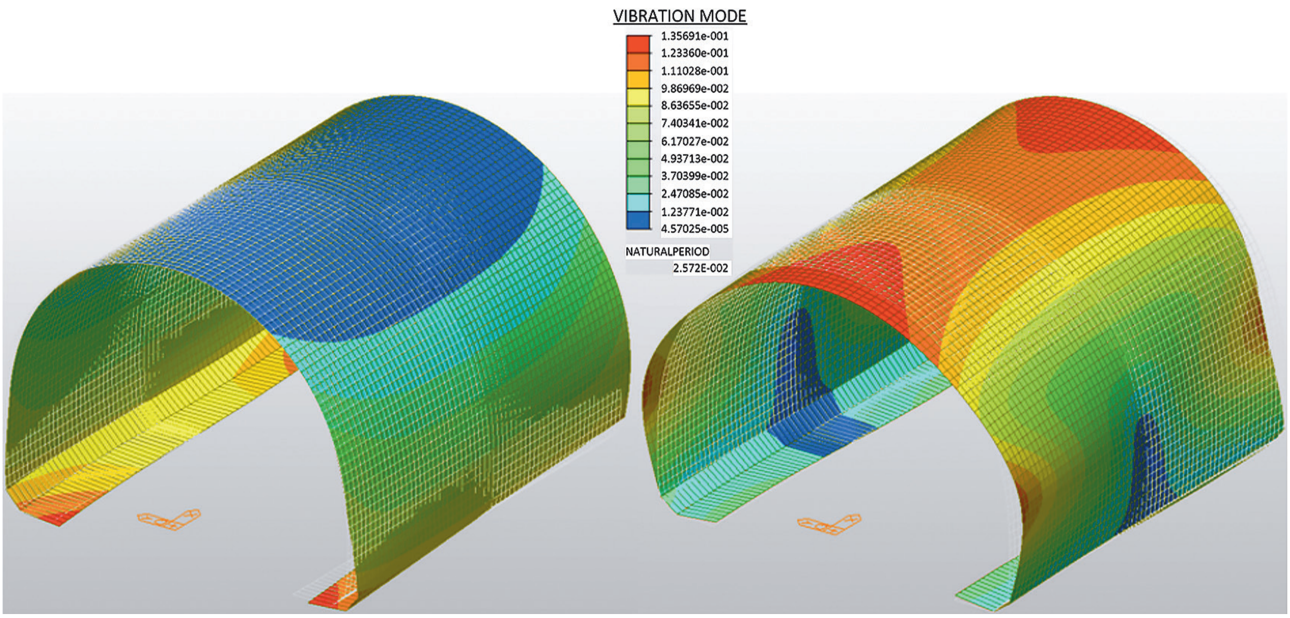

Fig. 15: Rotational oscillation of the secondary lining as a whole around X calculated by the subspace iteration method; left – eigenform (52,41 Hz), right – eigenform (88,30 Hz)

Table 2: Modal masses in relative scale, calculated by the subspace iteration method

Eigenform	Modal mass in the X direction	Modal mass in the Y direction	Modal mass in the Z direction	Modal mass torsion in X direction	Modal mass torsion in Y direction	Modal mass torsion in Y direction
	[%]	[%]	[%]	[%]	[%]	[%]
1	0	19,15	2,04	1,54	0	0
2	0	17,82	2,18	1,43	0	0
3	1,71	0	0	0	0,17	4,07
4	1,87	0	0	0	0,19	3,73
5	0,01	0	0	0	0	55,83
6	61,05	0	0	0	27,2	0,01
7	0	8,05	3,06	12,21	0	0
8	0	2,45	8,99	3,72	0	0
9	0	6,02	2,92	9,69	0	0
10	0	2,41	8,25	3,88	0	0
11	17,02	0	0	0	25,28	0,01
12	0,01	0	0	0	0,01	25,73
13	0	43,48	0	28,85	0	0
14	0	0	0	0	0	0
15	0	0	0	0	0	0
16	16,53	0	0	0	34,84	0
17	0	0	0	0	0	9,02
18	0,04	0	0	0	0,48	0,04
19	0,02	0	0	0	0,2	0,09
20	0	0	20,07	0	0	0
21	0	0,16	0	31,21	0	0
22	0	0	45,82	0	0	0
Sum	98,26	99,54	93,33	92,53	88,37	98,53

forces would not arise during a natural seismic event due to the structure's sufficient stiffness, as 38.88 Hz > 33 Hz. This is ensured by the subsoil's sufficient stiffness, the structure's shape, the lining's thickness and the material's quality.

Acknowledgements: This work was partially supported by the Slovak Research and Development Agency under contract VEGA 1/0180/24.

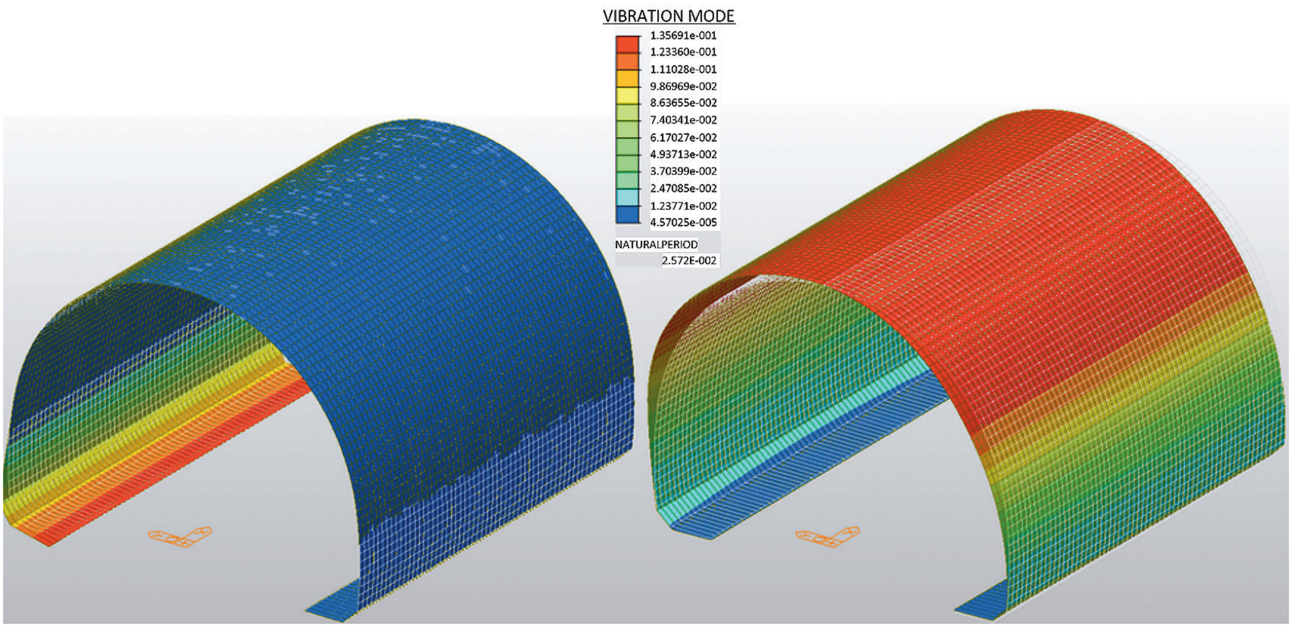

Fig. 16: Rotational oscillation of the secondary lining as a whole around Y calculated by the subspace iteration method; left – eigenform (38,88 Hz), right – eigenform (81,69 Hz)

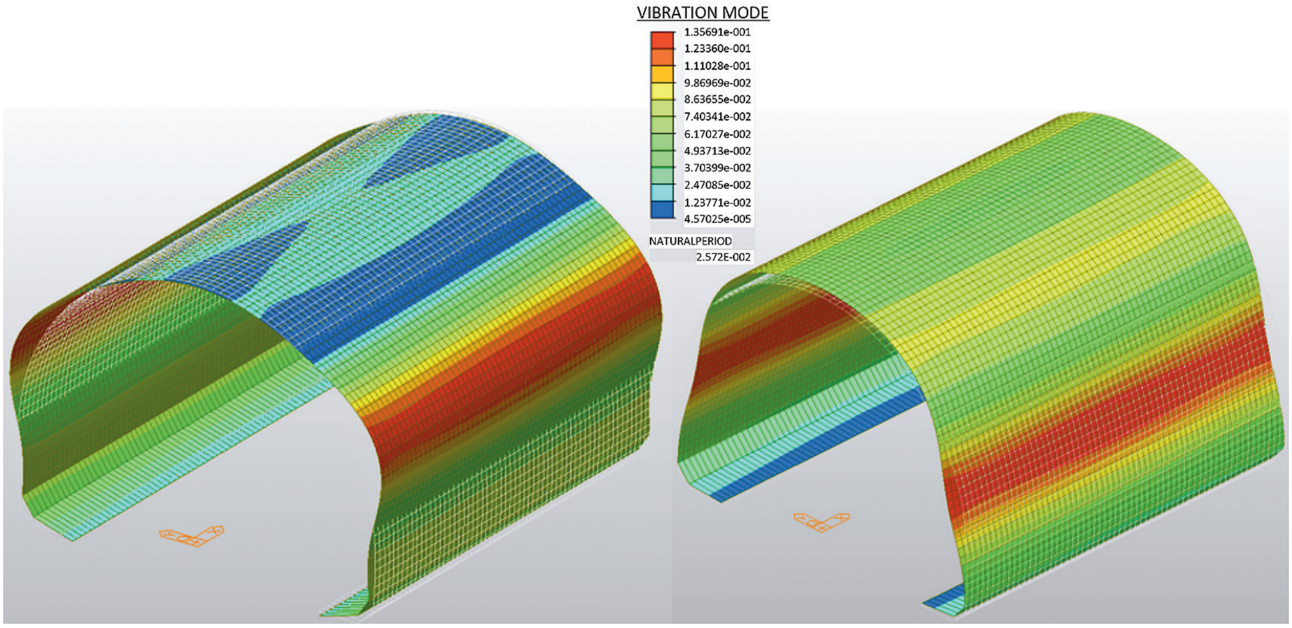

Fig. 17. Rotational oscillation of the secondary lining as a whole around Z calculated by the subspace iteration method; left – eigenform (93,54 Hz), right – eigenform (98,53 Hz).

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